Energy Room Impulse Response Prediction by an Extended Image Source/Hierarchical Radiosity Combined Method

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RESUMO: Este artigo incide sobre um método combinado para a previsão de respostas impulsivas energéticas em salas, cujas paredes reflectem a energia sonora sob componentes difusas e especulares. As componentes reflectidas especularmente são obtidas através de um método das imagens estendido, em que a reverberação tardia é calculada através de métodos estatísticos. As componentes reflectidas difusamente são calculadas por um método de radiosidade hierárquica dependente do tempo. Os dois métodos são aplicados de forma inteiramente independente. O único elo de ligação entre os dois é estabelecido através do coeficiente de difusividade das paredes.
São discutidas técnicas com vista à redução do tempo de cálculo relativamente ao método das imagens estendido.
São também abordados aspectos de implementação relativamente ao método de radiosidade hierárquica.
São apresentadas previsões obtidas com o novo método combinado, sublinhando-se a flexibilidade e os tempos de cálculo reduzidos.

ABSTRACT: This paper presents a combined method for the prediction of the energy impulse response in rooms where the boundaries are composed of surfaces with reflection characteristics that are a mixture of specular and diffuse components. The specularly reflected components are obtained by an extended image source method with statistical addition of late reverberation, whereas the diffusely reflected components are calculated by a time-dependent hierarchical radiosity method. Each of the methods is operated independently, with the diffusivity coefficients of the surfaces establishing the only link between them.
Accelerating techniques for the extended image source method will be portrayed and, regarding the hierarchical radiosity method, implementation issues will be highlighted.
Prediction results will be shown, and a discussion on the flexibility, accuracy and low computation times achieved by this new combined method will be given.

1. INTRODUCTION

The prediction of sound fields inside enclosures with walls of given impedance can be accomplished by solving the acoustic wave equation in its linearized form and adopting the corresponding boundary conditions. Solving the wave equation analytically is only possible for highly simplified geometries and idealized boundary conditions. Therefore, several methods such as FEM, BEM or FDTD have been developed in order to obtain numerical solutions for sound fields inside enclosures. However, the computation time taken by these methods is very high.
Another approach consists in using the geometrical acoustics theory, whereby approximate solutions in the time or frequency domain can be obtained.

For most practical purposes in room acoustics it is only necessary to obtain the prediction of how the sound energy propagates inside the enclosures. Therefore, the underlying calculations can be considerably simplified.
Recently, an effort has been made to include some wave theory aspects in the calculation of sound energy inside rooms using the geometrical acoustics theory. This has been done by adopting the idea that in order to model the complex reflection characteristics of surfaces one should consider that a fraction of energy incident on the surface is reflected specularly while the remaining fraction is reflected in a purely diffuse way (also known as lambertian reflection). Both contributions are determined by the so called diffusivity coefficient, which can be a wave length dependent quantity, having a value comprised between zero and one.
It will be show in this paper that if the diffusivity coefficient of the surfaces is small for a certain group of walls and near one for another group of walls, then a combined method can be used for solving the governing equations for the sound energy propagation inside the enclosures.
The combined method yields separately the specularly reflected sound components by an extended image source method and the diffusely reflected sound components using a radiosity based method.

2. THEORY

An equation of motion can be defined for the time dependent behaviour of the acoustic energy flux B reflected from the walls of an arbitrary enclosure (in a similar fashion as in [1] for purely lambertian enclosures):

$$\begin{aligned} B(\mathbf{s},t) = \iint_\Gamma \ldots \iint_\Gamma & B(\mathbf{s_0},0) W_{dt_1}^{0}\left[\mathbf{s_0} \rightarrow \mathbf{s_1} \mid \left(\mathbf{s_{-1}} \rightarrow \mathbf{s_0}\right)_0\right] \rho(\mathbf{s}_1) e^{-m\|\mathbf{s_1}-\mathbf{s_0}\|} W_{dt_2}^{dt_1}\left[\mathbf{s_1} \rightarrow \mathbf{s_2} \mid \left(\mathbf{s_0} \rightarrow \mathbf{s_1}\right)_{dt_1}^{0}\right] \\ & \rho(\mathbf{s}_2) e^{-m\|\mathbf{s_2}-\mathbf{s_1}\|} \ldots W_{dt_n}^{dt_{n-1}}\left[\mathbf{s_{n-1}} \rightarrow \mathbf{s} \mid \left(\mathbf{s_{n-2}} \rightarrow \mathbf{s_{n-1}}\right)_{dt_{n-1}}^{dt_{n-2}}\right] \rho(\mathbf{s}) e^{-m\|\mathbf{s_{n-1}}-\mathbf{s}\|} d\mathbf{s_0} d\mathbf{s_1} \ldots d\mathbf{s_{n-1}} \end{aligned} \quad (1)$$

where $t = dt_0 + dt_1 + \ldots + dt_n$ and Γ denotes the enclosure boundary.
$W_{dt_n}^{dt_{n-1}}\left[\mathbf{s_j} \rightarrow \mathbf{s_k} \mid \left(\mathbf{s_i} \rightarrow \mathbf{s_j}\right)_{dt_{n-1}}^{dt_{n-2}}\right]$ represents the transition amplitude that energy flux is exchanged from $\mathbf{s_j}$ to $\mathbf{s_k}$ during the time interval comprised between dt_{n-1} and dt_n, given that the energy flux present at $\mathbf{s_j}$ results from energy radiated from $\mathbf{s_i}$ at the previous time interval comprised between dt_{n-2} and dt_{n-1}. $\rho(\mathbf{s_i})$ equals the reflection coefficient of the wall element located at $\mathbf{s_i}$, considered as independent of the angle of incidence, and the exponential factors account for air attenuation. All of the above mentioned quantities are wave length dependent.
Equation (1) can be recast into operator form as:

$$B(\mathbf{s},t) = \Theta^k B(\mathbf{s},0) \quad (2)$$

where Θ is the following integral reflection and transport operator:

$$(\Theta B)(\mathbf{s},\tau+dt_n)=\iint_{\Gamma} B(\mathbf{s_{n-1}},\tau)W_{\tau+dt_n}^{\tau}\left[\mathbf{s_{n-1}}\to\mathbf{s}\,|\left(\mathbf{s_{n-2}}\to\mathbf{s_{n-1}}\right)_{dt_{n-1}}^{dt_{n-2}}\right]\rho(\mathbf{s})e^{-m\|\mathbf{s_{n-1}}-\mathbf{s}\|}d\mathbf{s_{n-1}} \tag{3}$$

If the reflection law of the boundary can be split into the sum of a purely diffuse reflected component and of a purely specular term:

$$\rho(\mathbf{s})=\rho_D(\mathbf{s})+\rho_S(\mathbf{s})=\rho(\mathbf{s})\Delta(\mathbf{s})+\rho(\mathbf{s})\left(1-\Delta(\mathbf{s})\right) \tag{4}$$

*Δ (***s***)* is the diffusivity coefficient for wall element **s**. Consequently, operator Θ is also divided into two components:

$$\begin{aligned}
&\Theta=\Theta_D+\Theta_S\\
&(\Theta_D B)(\mathbf{s},\tau+dt_n)=\iint_{\Gamma} B(\mathbf{s_{n-1}},\tau)W_{\tau+dt_n}^{\tau}\left[\mathbf{s_{n-1}}\to\mathbf{s}\right]\rho(\mathbf{s})\Delta(\mathbf{s})e^{-m\|\mathbf{s_{n-1}}-\mathbf{s}\|}d\mathbf{s_{n-1}}\\
&(\Theta_S B)(\mathbf{s},\tau+dt_n)=\iint_{\Gamma} B(\mathbf{s_{n-1}},\tau)W_{\tau+dt_n}^{\tau}\left[\mathbf{s_{n-1}}\to\mathbf{s}\,|\left(\mathbf{s_{n-2}}\to\mathbf{s_{n-1}}\right)_{\tau}^{\tau-dt_{n-1}}\right]\rho(\mathbf{s})(1-\Delta(\mathbf{s}))e^{-m\|\mathbf{s_{n-1}}-\mathbf{s}\|}d\mathbf{s_{n-1}}
\end{aligned} \tag{5}$$

Note that the transition amplitude term for the diffuse reflection and transport operator does not depend now on the previous transition history (e.g. the conditional probability ceases to exist because of the lambertian reflection law). Equation (2) then becomes:

$$B(\mathbf{s},\,t)=(\Theta_D+\Theta_S)^k\,B(\mathbf{s},\,0) \tag{6}$$

which can be expanded by using the binomial formula:

$$B(\mathbf{s},\,t)=\sum_{m=0}^{k}\left(\frac{k}{m}\right)\Theta_D{}^{m}\,\Theta_S{}^{k-m}\,B(\mathbf{s},\,0) \tag{7}$$

If one assumes that the value of the diffusivity factor Δ*(***s***)* is small in a certain part of the enclosure, denoted by S_L, and near one in another part of the enclosure, S_H, then only the "linear" terms of the above sum are of considerable magnitude [2]:

$$B(\mathbf{s},\,t)=(\Theta_D{}^k+\Theta_S{}^k)B(\mathbf{s},\,0) \tag{8}$$

Therefore, the motion of the energy flux inside this enclosure can be determined by solving independently for the diffuse operator equation and for the specular operator equation.

2.1 Solving the Specular Operator Equation

The ideally specular transition amplitude factor in equation (5) can be written as [3]:

$$W_{dt_n}^{dt_{n-1}}\left[\mathbf{s_{n-1}} \to \mathbf{s} \mid \left(\mathbf{s_{n-2}} \to \mathbf{s_{n-1}}\right)_{dt_{n-1}}\right] = \frac{\delta(\theta-\theta')\delta(\phi-\phi'\pm\pi)}{4\pi\left\|\mathbf{s_{n-1}}-\mathbf{s}\right\|^2} \qquad (9)$$

where θ is equal to the angle between the vector ($\mathbf{s_{n-2}}$-$\mathbf{s_{n-1}}$) and the normal vector at $\mathbf{s_{n-1}}$, $\boldsymbol{n}(\mathbf{s_{n-1}})$, and θ' is equal to the angle between the vector ($\mathbf{s}$-$\mathbf{s_{n-1}}$) and $\boldsymbol{n}(\mathbf{s_{n-1}})$. ϕ and ϕ' are the azimuth angles at the same point.

Due to the specular nature of the reflection factor, translated by the normalized Dirac distributions, the only contributions to the specular operator (5) must come from directions θ and ϕ such as given by the well known Snell-Descartes Law stating that the outgoing polar angle must be equal to the incident polar angle and that the outgoing direction must be contained in the plane of incidence. Therefore, one solution method for the specular operator equation can be achieved by the usage of an extended image source method.

2.2 Solving the Diffuse Operator Equation

Introducing the usual notation for the ideally diffuse transition amplitude factor [4], the diffuse operator term in (5) can be written as:

$$(\Theta_D B)(\mathbf{s},\tau) = \iint_{\Gamma} B\left(\mathbf{s_{n-1}}, \tau - \frac{\left\|\mathbf{s}-\mathbf{s_{n-1}}\right\|}{c}\right)\frac{\cos\theta\cos\theta'}{\pi\left\|\mathbf{s_{n-1}}-\mathbf{s}\right\|^2}\Delta(\mathbf{s})\rho(\mathbf{s})e^{-m\left\|\mathbf{s_{n-1}}-\mathbf{s}\right\|}vis(\mathbf{s_{n-1}},\mathbf{s})d\mathbf{s_{n-1}} \qquad (10)$$

where the arguments of the cosine functions are the same angles as defined previously in equation (9), and where a visibility function, taking values between 0 (complete occlusion) and 1 (complete visibility), was included in order to cope with non-convex enclosures.

Equation (10) can be solved by using a finite element approach by discretizing the boundary into M patches, and adopting a Galerkin method with constant basis functions:

$$\begin{aligned}
(\tilde{\Theta}_D B)(S_j,\tau) &= \sum_{i=1}^{M}\frac{1}{S_j}\iint_{S_iS_j} B\left(S_i, \tau - \frac{\left\|S_i-S_j\right\|}{c}\right)\Delta(S_j)\rho(S_j)e^{-m\left\|S_i-S_j\right\|}vis(S_i,S_j)\frac{\cos\theta_i\cos\theta_j}{\pi\left\|S_i-S_j\right\|^2}dS_i dS_j \\
&= \sum_{i=1}^{M} B\left(S_i, \tau - \frac{\left\|S_i-S_j\right\|}{c}\right)\Delta(S_j)\rho(S_j)e^{-m\left\|S_i-S_j\right\|}\frac{1}{S_j}\iint_{S_iS_j}\frac{\cos\theta_i\cos\theta_j}{\pi\left\|S_i-S_j\right\|^2}vis(S_i,S_j)dS_i dS_j \qquad (11) \\
&= \sum_{i=1}^{M} B\left(S_i, \tau - \frac{\left\|S_i-S_j\right\|}{c}\right)\Delta(S_j)\rho(S_j)e^{-m\left\|S_i-S_j\right\|}F(S_i \to S_j)
\end{aligned}$$

where $F(S_i \rightarrow S_j)$ is the geometric form factor between patch S_i and patch S_j.

This expression defines the time dependent acoustic radiosity equation, whose corresponding time independent form is well known and frequently used in the fields of Radiation Transfer and Computer Graphics.

3. IMPLEMENTATION

The combined method was implemented in a computer program.

The algorithm for solving the specular reflected components was implemented using an extended image source method, while the algorithm for solving the diffusely reflected components is based on a Hierarchical Radiosity [5] algorithm that allows the error thresholds to be defined by the user in order to guarantee maximum accuracy of the solution and low computation times.

3.1 Extended Image Source Method

Because of the exponential growth of the number of potential image sources with reflection order, given by:

$$M + M(M-1) + M(M-1)^2 + ... + M(M-1)^{K-1} = \frac{M(M-1)^K - 1}{M-2} \approx (M-1)^K \qquad (12)$$

M being the number of considered walls and K the reflection order, the exact calculation of image sources is done only until some given reflection order, typically 7. Information supplied by this phase is then used for the statistical and deterministic extrapolation to higher reflection orders in order to obtain the reverberation tail of the room's impulse response.

The extrapolation step resorts to the following parameters, obtained during the exact geometrical construction phase:

1. Number of visible images per reflection order: ***N(K)***
2. Mean distance to the receiver of visible images of order K: $\boldsymbol{\bar{D}(K)}$
3. Standard Deviation of the distance of visible images of order K: $\boldsymbol{\sigma(K)}$
4. Mean reflection coefficient for reflections of order K: $\boldsymbol{\bar{\rho}(K)}$
5. Standard Deviation of reflection coefficients of order K: ***g(K)***
6. Mean diffusivity coefficient for reflections of order K: $\boldsymbol{\bar{\Delta}(K)}$
7. Standard Deviation of the diffusivity coefficients of order K: $\boldsymbol{\eta(K)}$

The following functional relationship for the number of visible images per reflection order is adopted for the extrapolation step:

$$\lim_{K \to \infty} N(K) = a + bK + cK^2 \qquad (13)$$

where the coefficients *a*, *b* and *c* are obtained by a least-squares fit to the data obtained during the phase of exact geometrical construction of the visible images. The Statistical Distribution used for extrapolating the distances in function of reflection order *K* is determined by a Γ-distribution with shape parameter δ and scale parameter λ:

$$\begin{aligned} D(K) &= GammaDist[\delta(K),\lambda(K)] \\ \delta(K) &= \left(\frac{\bar{D}(K)}{\sigma(K)}\right)^2 \\ \lambda(K) &= \frac{(\sigma(K))^2}{\bar{D}(K)} \end{aligned} \tag{14}$$

N(K) reflections, as given by equation (13), are generated according to this distribution for every higher reflection order.

The mean reflection coefficient $\bar{\rho}(K)$, and the corresponding standard deviation *g(K)*, for reflections of order *K* are calculated from the registered collision frequencies with the walls, *f*:

$$\begin{aligned} \bar{\rho}(K) &= \sum_{j=1}^{M}\left[\frac{f(W_j,K)}{\sum_{i=1}^{M} f(W_i,K)}\right]\rho(W_j) \\ g(K) &= \overline{\rho^2(K)} - \left(\bar{\rho}(K)\right)^2 \end{aligned} \tag{15}$$

and the mean specularity coefficient, $\overline{1-\Delta(K)}$, and the corresponding standard deviation *η(K)*, for reflections of order *K*, are also calculated from the registered walls' collision frequencies:

$$\begin{aligned} \overline{1-\Delta(K)} &= \sum_{j=1}^{M}\left[\frac{f(W_j,K)}{\sum_{i=1}^{M} f(W_i,K)}\right](1-\Delta(W_j)) \\ \eta(K) &= \overline{(1-\Delta(K))^2} - \left(\overline{1-\Delta(K)}\right)^2 \end{aligned} \tag{16}$$

The room impulse response from specular reflections is thereby calculated from:

$$\begin{aligned} I_{specular}(t) &= \sum_{K=0}^{\text{maxgeoorder}} \sum_{i=1}^{N(K)} \frac{\Pi}{4\pi d_i^2}\rho_i e^{-md_i}(1-\Delta_i)\delta\left(t-\frac{d_i}{c}\right) \\ &+ \sum_{K=\text{maxgeoorder}+1}^{\text{maxorder}} \sum_{i=1}^{N(K)} \frac{\Pi}{4\pi D_i(K)^2}\bar{\rho}(K)e^{-mD_i(K)}\overline{1-\Delta(K)}\delta\left(t-\frac{D_i(K)}{c}\right) \end{aligned} \tag{17}$$

where "maxgeoorder" means the maximum reflection order for which the exact geometrical calculation of the visible images is done.

3.2 Hierarchical Time Dependent Radiosity Method

Solving the diffuse reflection operator by a finite element approach, as defined by equation (11), brings two problems:

> Problem 1: For the method of images it is required that the walls constituting the geometry of the enclosure be as large as possible, whereas for the finite element approach one desires that the input polygons constitute a mesh of several smaller "patches".

> Problem 2: If M initial walls are split into n patches, then the number of energy links, i.e. the approximate number of form factors, will be proportional $O(n^2)$, and therefore the calculation effort is high.

One solution for both problems is to adopt a Multi-Resolution approach through hierarchical linking [5], whereby the patches are generated adaptively from the input walls and the number of links (form factors) is $O(M^2+n)$, therefore permitting a considerable computational saving.
The hierarchical subdivision stop criteria is based in first place on the absolute minimum area of patches (defined by the user) and in second place on a threshold condition for the form factors between pairs of patches (also defined by the user).

Form factors are calculated by using the analytic formula of the form factor from a differential area to a parallel disc, and the
visibility factor for the occluded form factor is calculated by ray-casting.
The solution of the time-dependent equations is done by a

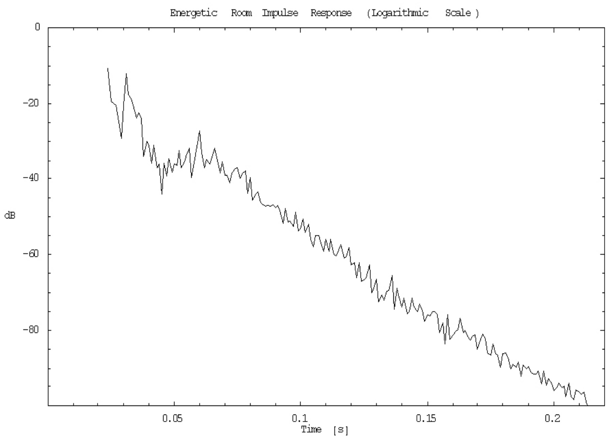
Gauss-Seidel relaxation scheme, using an
energy "Push-pull" operation throughout each hierarchy of patches.

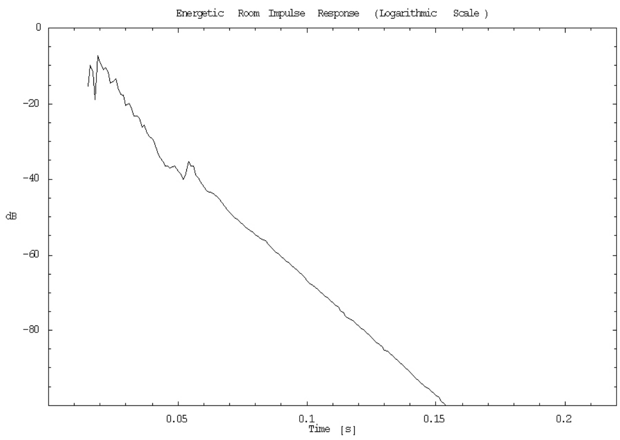
In order to limit the exponential growth of diffuse reflections with increasing reflection orders, a condensing algorithm with an internal sampling rate is used.

At each iteration, energy is gathered at predefined receiving points, calculated through the well known lambertian cosine law.

4. RESULTS

The combined method is composed of two methods for solving for the energy propagation inside enclosures of arbitrary geometries and having walls with materials having different reflection coefficients and diffusivity coefficients. Both algorithms run independently from each other, the only "connection" being made through the diffusivity coefficients. The total room impulse response is obtained by the sum of the specular response and the diffuse response. Prediction results are shown in figures 1, 2 and 3.





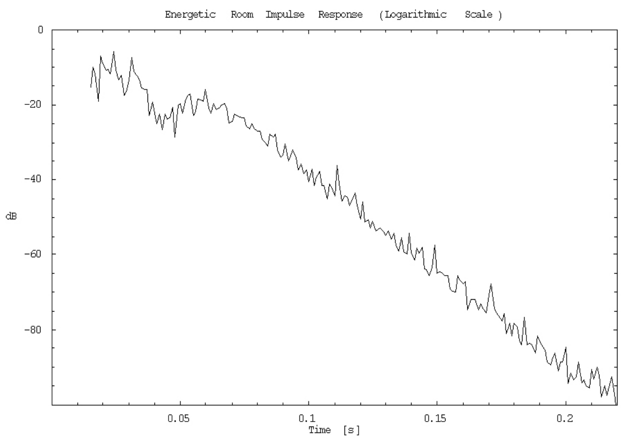


Figure 1 - *Specular Impulse Response* Figure 2 – *Diffuse Impulse Response*
Figure 3 – *Total Impulse Response*

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