SMART CLUSTER CONTROL OF PLANAR STRUCTURES

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Nobuo Tanaka
Department of Mechanical Engineering
Tokyo Metropolitan Institute of Technology
6-6 Asahigaoka, Hino-shi, Tokyo 191-0065, Japan
Tel: +81-42-585-8668
Fax: +81-42-583-5119
E-mail: ntanaka@cc.tmit.ac.jp

ABSTRACT
This paper proposes a novel active vibration control methodology termed “cluster control” that permits the suppression of all structural modes belonging to the cluster of interest. First, it is mathematically verified that every eigenfunction of a two-dimensional structure can be expressed in a form of the “(*m,n*) modes”, *m* and *n* being modal indices. Then, a cluster filtering method is introduced, whereby all eigenfunctions are divided into a finite number of groups (clusters), each member of the group retaining the same characteristics in common. By verifying the principle of reciprocity between sensing and actuation of a cluster, a cluster actuation method is then introduced, enabling excitation of the eigenfunctions of a targeted cluster without affecting any repercussion on the other clusters. Moreover the cluster control method consisting of both the cluster filtering and actuation is presented, allowing suppression of all structural modes without causing instability of the control system. Finally, an experiment demonstrates the validity of the proposed method for suppressing all vibration modes of a planar structure.

1. INTRODUCTION
Difficulty in actively controlling the vibration of a distributed-parameter structure attributes to an infinite number of structural modes it has. When attempting to apply a conventional active vibration control method using point sensors/actuators for a distributed-parameter structure, instability of a control system is likely to occur due to an incomplete description of the mathematical model of the distributed-parameter structure caused by residual modes truncated in process of constructing the mathematical model. To cope with this problem, several methods have been presented — introduction of collocation scheme, utilization of observer, application of modal filtering and comb filtering.

It is true that a distributed-parameter structure possesses an infinite number of structural modes, however, it is prone to be overseen a fact that an infinite number of structural modes can be assembled into a finite number of groups (clusters) possessing the same properties in common. For instance, the eigenfunctions of a planar structure — determined by boundary conditions of a structure — can be denoted as the (*m, n*) modes, *m* and *n* being modal indices, which are further placed into four clusters; the odd/odd modal cluster, odd/even modal cluster, even/odd modal cluster and even/even modal cluster. With the common attributes each cluster has, it is expected to achieve a significant control effect with a simple control strategy owing to the common properties, and thus the paper calls this a “cluster control”.

With a view to construing the cluster control, it is worth introducing a “power mode” [1~4] that plays a key role in the sound radiation of a structure— a common thread between a vibration field and an acoustic field. The power mode can be placed into the aforementioned four categories; among the four, the odd/odd modes comprise the first power mode, the greatest contributor to the total acoustic power radiated from a vibrating structure. Therefore, when attempting to reduce the sound radiation of a structure, it is suffice to suppress the specific modal cluster — the odd/odd modal cluster — rather than trying to suppress all structural modes at the expense of energy and cost. On the contrary, without considering the power mode if one tries to actively control the structure-borne sound, the control is likely to fail — the phenomenon that a sound level increases while a vibration level decreases is possible in theory, and often encountered in practice.

In light of this, this paper proposes a novel active vibration control methodology termed “cluster control”, which enables to suppress all structural modes belonging to the targeted cluster with a significantly simple control method. First, it is mathematically verified that the eigenfunctions of a planar structure can be expressed as the (m, n) mode using modal indices *m* and *n*. Then, a cluster filtering method is introduced, whereby all eigenfunctions of a distributed-parameter structure are placed into a finite number of clusters, each member of a cluster possessing the same characteristics in common. By verifying the reciprocity between sensing and actuation of a cluster, a cluster actuation method is then introduced, enabling the excitation of only the targeted cluster of a structure. Moreover the cluster control method consisting of both the cluster filtering and actuation is presented, allowing suppression of all structural modes without causing instability of the control system. Finally, an experiment demonstrates the validity of the cluster control for suppressing all vibration modes of a planar structure.

2. CLUSTER CONTROL

2.1 Cluster filtering

Consider a partial differential equation of motion of an undamped planar structure

$$\mathbf{L}[w(\mathbf{r},t)] + m(\mathbf{r})\ddot{w}(\mathbf{r},t) = f(\mathbf{r},t) \quad (1)$$

where $w(\mathbf{r},t)$ is the deflection of the structure at $\mathbf{r} = (x,y)$, $\dot{}$ the time derivative, $m(\mathbf{r})$ the mass density, $f(\mathbf{r},t)$ the distributed force, and $\mathbf{L}$ a self-adjoint differential operator. Using the eigenfunctions $\varphi_k(\mathbf{r})$ and modal coordinates, η_k and f_k ($k = 1, 2, 3\cdots$), $w(\mathbf{r},t)$ and $f(\mathbf{r},t)$ may be expressed as

$$w(\mathbf{r},t) = \sum_{k=1}^{\infty} \varphi_k(\mathbf{r})\eta_k(t) \quad (2)$$

$$f(\mathbf{r},t) = \sum_{k=1}^{\infty} m(\mathbf{r})\,\varphi_k(\mathbf{r})\,f_k(t) \quad (3)$$

Consider a velocity sensor placed at an arbitrary position $\mathbf{r}_i$ of a rectangular plate of dimensions $L_x \times L_y \times h$. Then the sensor output $v(\mathbf{r}_i, t)$ is expressed in terms of eigenfunctions $\varphi_k(\mathbf{r}_i)$ and modal coordinates ,

$$v(\mathbf{r}_i,t) = \sum_{k=1}^{\infty} \varphi_k(\mathbf{r}_i)\dot{\eta}_k(t) \quad (4)$$

Next, place four velocity sensors at the corner of the panel such that the following conditions are satisfied

$$\mathbf{r}_1 = (x_1,y_1) \quad (5)$$

$$\mathbf{r}_2 = (L_x - x_1,y_1) \quad (6)$$

$$\mathbf{r}_3 = (L_x - x_1,L_y - y_1) \quad (7)$$

$$\mathbf{r}_4 = (x_1,L_y - y_1) \quad (8)$$

where

$$x_1 << L_x, \qquad y_1 << L_y \quad (9)$$

Moreover, consider the case when the eigenfunctions categorized into four kinds satisfy the properties of symmetry or asymmetry yielded as follows:

$$\varphi^{o/o}(\mathbf{r}_1) = \varphi^{o/o}(\mathbf{r}_2) = \varphi^{o/o}(\mathbf{r}_3) = \varphi^{o/o}(\mathbf{r}_4) \quad (10)$$

$$\varphi^{o/e}(\mathbf{r}_1) = \varphi^{o/e}(\mathbf{r}_2) = -\varphi^{o/e}(\mathbf{r}_3) = -\varphi^{o/e}(\mathbf{r}_4) \quad (11)$$

$$\varphi^{e/o}(\mathbf{r}_1) = -\varphi^{e/o}(\mathbf{r}_2) = -\varphi^{e/o}(\mathbf{r}_3) = \varphi^{e/o}(\mathbf{r}_4) \quad (12)$$

$$\varphi^{e/e}(\mathbf{r}_1) = -\varphi^{e/e}(\mathbf{r}_2) = \varphi^{e/e}(\mathbf{r}_3) = -\varphi^{e/e}(\mathbf{r}_4) \quad (13)$$

Then, in light of Eq. (10) ~ (13), combining all of the sensor output leads to

$$\begin{aligned} &v(\mathbf{r}_1,t) + v(\mathbf{r}_2,t) + v(\mathbf{r}_3,t) + v(\mathbf{r}_4,t) \\ &= \left\{\varphi_1(\mathbf{r}_1)+\varphi_1(\mathbf{r}_2)+\varphi_1(\mathbf{r}_3)+\varphi_1(\mathbf{r}_4)\right\}\dot{\eta}_1(t) \\ &+ \left\{\varphi_2(\mathbf{r}_1)+\varphi_2(\mathbf{r}_2)+\varphi_2(\mathbf{r}_3)+\varphi_2(\mathbf{r}_4)\right\}\dot{\eta}_2(t) \\ &\cdots \\ &= 4\varphi_1^{o/o}(\mathbf{r}_1)\dot{\eta}_1^{o/o}(t) + 4\varphi_2^{o/o}(\mathbf{r}_1)\dot{\eta}_2^{o/o}(t) + 4\varphi_3^{o/o}(\mathbf{r}_1)\dot{\eta}_3^{o/o}(t) + \cdots \end{aligned} \quad (14)$$

where $\varphi^{o/o}$ and $\eta_i^{o/o}$ denote the *i*th odd/odd modal eigenfunction and the associated modal coordinate, respectively. Then, it is immediately apparent that the mere combination of the outputs of the four sensors placed for satisfying Eq. (10) enables the extraction of the odd/odd modes. Similarly, summing up the sensor outputs by adjusting its polarities in accordance with the properties of the eigenfunctions, the combined sensor outputs are given by

$$v(\mathbf{r}_1,t) + v(\mathbf{r}_2,t) - v(\mathbf{r}_3,t) - v(\mathbf{r}_4,t) = 4\varphi_1^{o/e}(\mathbf{r}_1)\dot{\eta}_1^{o/e} + 4\varphi_2^{o/e}(\mathbf{r}_1)\dot{\eta}_2^{o/e} + 4\varphi_3^{o/e}(\mathbf{r}_1)\dot{\eta}_3^{o/e} + \cdots \quad (15)$$

$$v(\mathbf{r}_1,t) - v(\mathbf{r}_2,t) - v(\mathbf{r}_3,t) + v(\mathbf{r}_4,t) = 4\varphi_1^{e/o}(\mathbf{r}_1)\dot{\eta}_1^{e/o}(t) + 4\varphi_2^{e/o}(\mathbf{r}_1)\dot{\eta}_2^{e/o}(t) + 4\varphi_3^{e/o}(\mathbf{r}_1)\dot{\eta}_3^{e/o}(t) + \cdots$$

$$v(\mathbf{r}_1,t) - v(\mathbf{r}_2,t) + v(\mathbf{r}_3,t) - v(\mathbf{r}_4,t) = 4\varphi_1^{e/e}(\mathbf{r}_1)\dot{\eta}_1^{e/e}(t) + 4\varphi_2^{e/e}(\mathbf{r}_1)\dot{\eta}_2^{e/e}(t) + 4\varphi_3^{e/e}(\mathbf{r}_1)\dot{\eta}_3^{e/e}(t) + \cdots \quad (17)$$

Thus, mere addition or subtraction of the sensor outputs allows partition of an infinite number of eigenfunctions into four clusters, and hence the work calls the signal processing a "cluster filtering".

Consider the characteristics of the four kinds of clusters. First, the odd/odd modal cluster comprising the first acoustic power mode æ the greatest contributor to the total acoustic power radiated from a vibrating structure æ characterizes the motions of a vibrating surface around the sensor locations, $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ and $\mathbf{r}_4$ — they are all in phase with the origin at $\mathbf{r}_1$, i.e., denoted as {+, +, +, +}. Likewise, {+, +, -, -} for the odd/even modal cluster, {+, -, -, +} for the even/odd modal cluster and {+, -, +, -} for the even/even modal cluster. With this in mind, constraint on the placement of control actuators will be considerably loosened to meet collocation[1]; the collocation holds if the motion of a vibrating nodal area with a sensor attached satisfies the aforementioned phase properties, thereby allowing control actuators to be placed inside any of the nodal areas.

2.2 Cluster Actuation

Replace the sensors used for cluster filtering with actuators. Then the exciting force f($\mathbf{r}$,t) is written as

$$f(\mathbf{r},t) = \sum_{l=1}^{4} F_l(t)\,\delta(\mathbf{r} - \mathbf{r}_l) \quad (18)$$

where δ denotes a Dirac delta function. Substituting Eqs. (18) and (2) into Eq. (1), multiplying the resultant Eq. (1) by $\varphi_i(\mathbf{r})$ and integrating it over the domain D, the equation of motion in Eq. (1) may be expressed in a form of a modal coordinate system,

$$\ddot{\eta}_i(t) + \omega_i^2\eta_i(t) = \sum_{l=1}^{4} F_l(t)\,\varphi_i(\mathbf{r}_l) \quad (19)$$

Consider the excitation on the odd/odd modal cluster by four point actuators. For this, the polarities of the forces are determined in exactly the same way as was done in the cluster filtering,

$$F_1(t) = F_2(t) = F_3(t) = F_4(t) \quad (20)$$

By substituting Eq. (20) into Eq. (19) and taking into consideration the properties of eigenfunctions given in Eqs. (10), Eq. (19) becomes

$$\ddot{\eta}_i(t) + \omega_i^2\eta_i(t) = \begin{cases} 4F_1(t)\,\varphi_i^{o/o}(\mathbf{r}_1) & \text{if } i \text{ is the odd / odd modes} \\ 0 & \text{if } i \text{ is not the odd / odd modes} \end{cases} \quad (21)$$

Clearly from Eq. (21), only the odd/odd modal cluster is excited, the other modal clusters remaining intact. Thus, merely driving four actuators simultaneously with the same polarities permits the excitation on only the odd/odd modes.

Similarly, by setting the signs of the forces as

$$F_1(t) = F_2(t) = -F_3(t) = -F_4(t) \quad (22)$$

$$F_1(t) = -F_2(t) = -F_3(t) = F_4(t) \quad (23)$$

$$F_1(t) = -F_2(t) = F_3(t) = -F_4(t) \quad (24)$$

and driving those forces simultaneously, the excitations on the odd/even, even/odd and even/even modal cluster become possible, respectively. As such, the force driving method described above enables the excitation on the targeted cluster without causing any influence on the other clusters, and hence this paper calls this excitation method a "cluster actuation". Note that the cluster actuation is derived by simply replacing the sensors for the cluster filtering with actuators, so that these methods are in the relation of reciprocity.

2.3 Cluster control

Difficulty in controlling the vibration of a distributed-parameter structure lies in an infinite number of eigenfunctions (structural modes) it has. To overcome the problem, this paper presents a "cluster control" consisting of both the cluster filtering and cluster actuation. Applying the cluster filtering enables to place those structural modes into a finite number of clusters, each cluster possessing the same properties in common, while employing the cluster actuation permits to excite the targeted cluster independently. Thus, using both the cluster filtering and actuation, the cluster control avoids observation / control spillover destabilization of a feedback control system, thereby achieving a significant control effect with a simple control approach, e.g., direct feedback with high gain.

Unlike a conventional modal control approach using point sensors and point actuators mainly aiming at suppressing structural modes, the cluster control aims at suppressing a cluster of interest, thereby suppressing all the structural modes belonging to the cluster.

With a view to giving further insight into the significance of the cluster control, it is worthwhile using a specific example of the cluster control on the odd/odd modal cluster as a test vehicle. First, to construct the cluster control system of a planar structure, four sensors for cluster filtering and four actuators for cluster actuation are needed. The output signal of a cluster filtering on the

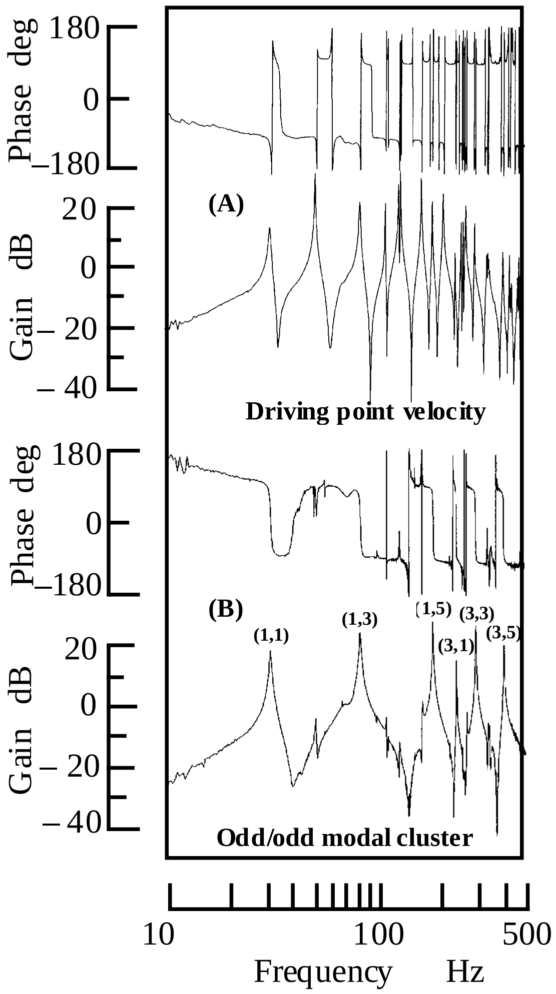

Fig. 1 Cluster filtering results for the odd/odd modes (A) Driving point velocity, (B) Odd/odd modal cluster

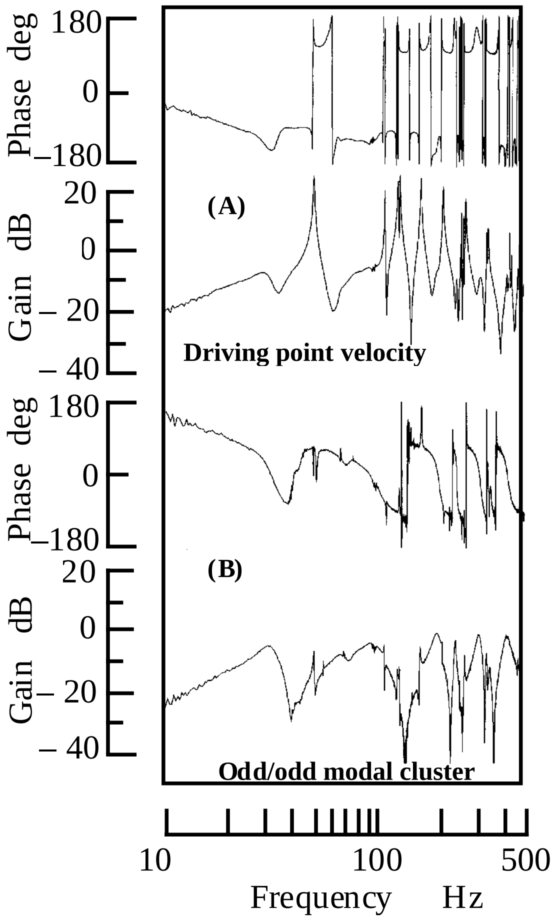

Fig. 2 Cluster control effect for suppressing the odd/odd modal cluster (A) Driving point velocity, (B) Odd/odd modal cluster

odd/odd modes, $e_{o/o}$, is described by

$$e_{o/o}(t) = \sum_{i=1}^{4} \dot{w}(\mathbf{r}_i,t) = \sum_{i=1}^{4} \sum_{k=1}^{\infty} \varphi_k(\mathbf{r}_i)\dot{\eta}_k(t)$$

$$= 4 \sum_{k=1}^{\infty} \varphi_k^{o/o}(\mathbf{r}_1)\dot{\eta}_k^{o/o}(t) \qquad (25)$$

Then, using Eq. (25) as a feedback control signal weighted with the gain of $g_{o/o}$, the control force is given by

$$f(\mathbf{r},t) = -4\, g_{o/o} \sum_{k=1}^{\infty} \varphi_k^{o/o}(\mathbf{r}_1)\dot{\eta}_k^{o/o}(t) \sum_{i=1}^{4} F_i \delta(\mathbf{r} - \mathbf{r}_i) \qquad (26)$$

Introducing the force polarities as shown in Eq. (20), multiplying Eq. (26) by $\varphi_i(\mathbf{r})$, integrating it over the domain D and substituting the resultant control force, Eq. (26), into Eq. (1), the equation of motion of a distributed-parameter structure is expressed in a form of a modal coordinate system as

$$\ddot{\eta}_i(t) + \omega_i^2 \eta_i(t) = \begin{cases} -16 g_{o/o} \sum_{k=1}^{\infty} \varphi_k^{o/o}(\mathbf{r}_1)\dot{\eta}_k^{o/o}(t)\, \varphi_i(\mathbf{r}_1) & i \text{ is odd/odd} \\ 0 & \text{if } i \text{ is not odd/odd} \end{cases} \qquad (27)$$

Clearly from Eq. (27), the output signal from the odd/odd modal cluster is fed back into only the odd/odd modal cluster, thereby avoiding the control / observation spillover[1] from a viewpoint of cluster.

In exactly the same way, the cluster control on the other clusters is performed by introducing the polarities of the sensors and actuators.

3. EXPERIMENT OF CLUSTER CONTROL

3.1 Cluster filtering

A steel rectangular panel with dimensions 88 cm x 180 cm x 9 mm is fixed on top of the enclosure constructed from 10 cm thick concrete walls. The uppermost edges are each of a “knife-blade” type in order to simulate simply supported boundary conditions. The enclosed cavity is packed with glasswool to dampen acoustic resonances. An electrodynamic shaker attached at $\mathbf{r}_a$ = (24 cm, 25 cm) provides the panel with an impact force as disturbance, while four point velocity sensors used for cluster filtering are mounted at $\mathbf{r}_1$ = (20 cm, 21 cm), $\mathbf{r}_2$ = (68 cm, 21 cm), $\mathbf{r}_3$ = (68 cm, 159 cm) and $\mathbf{r}_4$ = (20 cm, 159 cm). Taking into account the frequency spectrum of the impact force rolling off at 500 Hz, the maximum frequency for the cluster filtering is set to be 500 Hz, the modal frequencies and mode numbers in the frequency range up to 500 Hz— 22 structural modes exist —being listed in Table 1. Figure 2 (A) shows the driving point velocity (mobility), a ratio of velocity of the panel at the driving point $\mathbf{r}_a$ to the associated driving force, while Fig. 2 (B) illustrates it in terms of the cluster filtering on the odd/odd modes — obtained by merely combining all of the sensor outputs with the same polarity.

Clearly from Table 1, although there are 7 odd/odd modes in the frequency range of interest, the (1,7) mode is slightly excited as shown in Fig. 2 (B). This is due to the shaker location that happened to coincide with one of the nodal lines of the (1,7) mode.

3.2 Cluster control

Based upon both the cluster filtering and cluster actuation, the cluster control achieves the suppression of all the structural modes belonging to the cluster of interest. To construct the cluster control system for the odd/odd modal cluster, for instance, a cluster filtering output $e_{o/o}$ is directly fed back into four point actuators with an appropriate feedback gain $g_{o/o}$— determined by trial and error while observing the waveforms of impact vibration being suppressed. Thus the cluster control allows one to pursue the effective performance in a primitive manner owing to the intrinsic simplicity of the cluster control.

Figure 2(B) illustrates the control effect for suppressing the odd/odd modal cluster, the corresponding driving point velocity being depicted in Fig. 2 (A). Comparison of Fig. 2 (A) and Fig. 1(B) showing the driving point velocity before control reveals that only the targeted 7 odd/odd modes among 22 in the frequency band of interest are suppressed, while the other modes remain unaffected.

Control effect for suppressing all the clusters is shown in Fig. 3(A), and the corresponding cluster sensor outputs are illustrated in (B) through (E). In this case, all the clusters are treated equally, so that the weights on each cluster are all one. The suppression of all the clusters is equivalent to suppression of all the structural modes, and so all the resonant peaks in Fig. 3(A) are significantly suppressed.

4. CONCLUSIONS

An active vibration control of a distributed-parameter structure — a "cluster control" — has been presented, elucidating the capability of the cluster control for suppressing all the structural modes the cluster possesses. First, it is shown that eigenfunctions determined by boundary conditions of a planar structure may be expressed as the (m, n) mode using modal indices, *m* and *n*. Based upon the description of structural modes, a cluster filtering method is proposed, revealing that an infinite number of structural modes are placed into four clusters with the same properties in common, i.e., the odd/odd modal cluster, the odd/even modal cluster, the even/odd modal cluster and the even/even modal cluster. It is also shown that, by replacing the point sensors used for the cluster filtering with point actuators, a cluster actuation is found possible, enabling the excitation on only the cluster of interest without causing any affect on the other clusters, and hence the reciprocity principle holds between the cluster filtering and the cluster actuation. Then, using both the cluster filtering and the cluster actuation, a novel active vibration control method of a distributed-parameter structure, a "cluster control", is proposed, enabling one to control the targeted cluster without causing any repercussion on the other clusters. This method is particularly useful for controlling the power mode — dominant factor of structure-borne sound. Finally, using a simply supported thin panel, an experiment is conducted, demonstrating the validity of the cluster filtering, the cluster actuation and the cluster control.

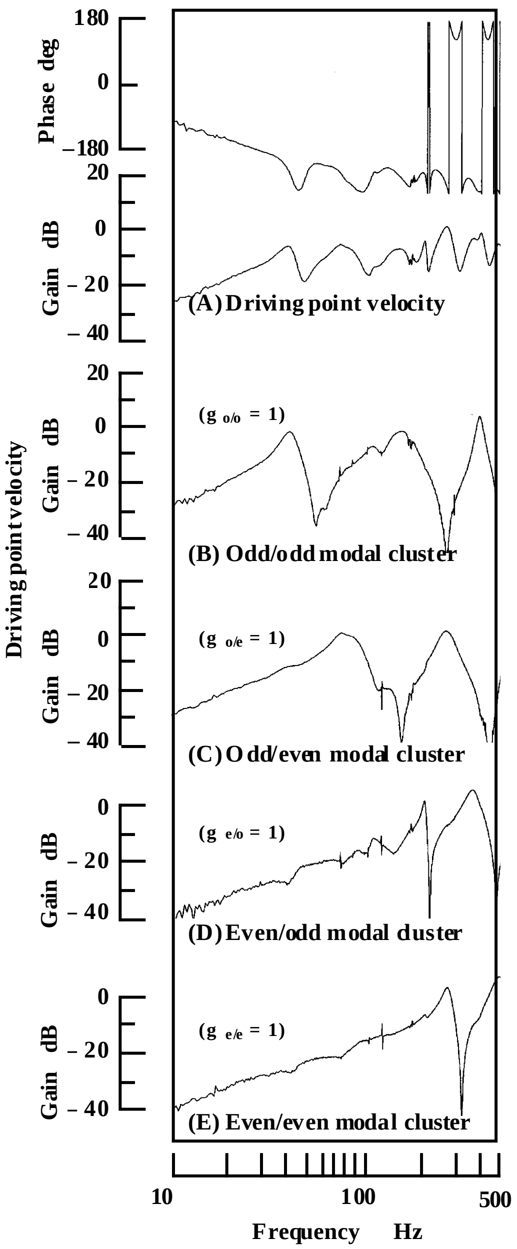

Fig.3 Cluster control effect for suppressing all the clusters

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