SIMULATION OF SOUND WAVE PROPAGATION IN TURBULENT FLOWS USING A LATTICE-BOLTZMANN SCHEME

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Abstract

The propagation of sound waves in instationary flows is often computed in two steps, which are the determination of the instationary flow field and calculation of sound propagation in the flow. Here, a "all-in-one" approach is proposed which is based on the Lattice-Boltzmann method. Two applications are presented: The solution of a benchmark problem which is compared to the analytical solution and a two-dimensional cut through an ultrasound gas flow meter. For the benchmark problem excellent agreement between theory and numerical results was obtained. For the ultrasound gas flow meter, the results clearly indicate the feasibility of such simulations. The measured transit times of a test signal agree well with rough estimations, furthermore they exhibit a lot of details which are related to the instationary flow field.

1 Introduction

Flow acoustic problems usually can be split into three more or less separate parts: The determination of the turbulent flow itself, the propagation of sound waves within a turbulent flow and in many cases the generation of sound by the flow. The interaction between this parts is weak or at least only one-way for most problems at low MACH-numbers. This means, that the turbulent flow generates sound waves, but this effect only plays a minor role with the determination of the instationary flow field itself. Also flow structures such as eddies in a turbulent flow modify the propagation of sound waves, but not vice versa. Therefore the simulation of such problems is often done in two or three steps: First the flow is calculated solving the Navier-Stokes equations, then the sound generation is computed with another model (acoustic analogies etc.) and finally the propagation effects are accounted for using a third model (ray tracing or solution of the convected wave equation). The reasons for doing so instead of using a "all-in-one-model" are the length disparities between the size of flow structures and the acoustic wave lengths and the precision of the flow solvers, which tend to be "noisy". This means that the numerical error is too large to gather information on the small scale of variability associated with sound waves. On the other hand, the series connection of this different models poses a lot of technical problems and the errors at each stage of the modeling process may add up inauspiciously.

Two of the problem parts described above namely the determination of the turbulent flow and sound propagation through the flow occur when the velocity of a flow is to be measured by the transit time of sound wave through a test section of known length. The transit time is effected by the convection of the sound waves and is measured as a phase difference between the transmitted and received signal, where signals of constant frequencies are used. This principle is often used in ultrasound gas flow meters. Here the size of the strongest eddies is of the same order of magnitude as wave length of the acoustic wave used for the measurement. Therefore a model which computes the instationary flow and the propagation of the test sound in one step is useful because the variability of the measured transit time is sought, which is introduced by the turbulence. In a model where all physical data are at hand at the same time the correlations between certain parameters can be tracked most easily.

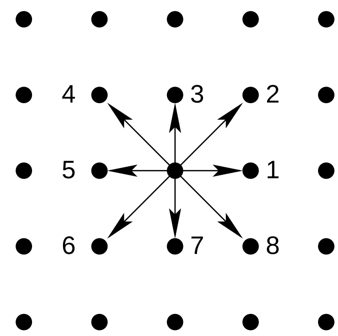

Figure 1: The D2Q9-Lattice-Boltzmann model is defined on a two dimensional orthogonal grid. The vectors $\Delta\vec{x}_i$ point from one node to the i-th of the 8 nearest neighbors. Together with zero velocity this gives 9 possible dynamic states.

The Lattice-Boltzmann method is a relatively new technique which was introduced for solving fluid dynamic problems up to moderate subsonic MACH-numbers. It evolved during the last two decades from the so-called Lattice-Gas method, which simulate the behavior of single fluid particles on a lattice. Since then the Lattice-Boltzmann method was successfully applied to problems which involve highly turbulent flows and extremely complex geometries, such as the flow around cars or through porous media [1]. Only very few studies dealt with the ability of the method to simulate sound wave propagation [2, 3, 4]. Here a first approach is presented to simulate convection effects on sound wave propagation with a Lattice-Boltzmann scheme.

2 Theory

The basic idea of the Lattice-Boltzmann method is to discretize the phase space of all the particles that make up the fluid to the effect that only a finite number of velocities and positions remains, which the particles are demanded to take at each tick of a clock counting the time in intervals of constant duration. The possible positions are given by the nodes of a grid with sufficient symmetry, and the possible velocities result from the restriction, that particles may be at rest or move from one lattice site to one of its nearest neighbors during one time interval. In this study an orthogonal grid is used in which the state of a lattice site (node) at the time $t+\Delta t$ is dependent on the state of the 8 sites in the direct neighborhood and the site itself at time t. On this lattice the vectors $\Delta\vec{x}_i$ are defined which point from one node to its nearest neighbor in the i-th direction (see fig. 1). The possible velocities are thus given by

$$\vec{v}_i = \frac{\Delta\vec{x}_i}{\Delta t}, \ \ i = 0\ldots 8$$

which includes particles at rest with $|\Delta\vec{x}_0| = 0$. In the following the index i which indicates the direction of the nearest neighbor site always ranges from $0\ldots 8$.

The Lattice-Boltzmann method is used to calculate particle flows between lattice sites. The flow $f_i(\vec{x}+\Delta\vec{x}_i, t+\Delta t)$ of particles that travel at time $t+\Delta t$ from a node at position $\vec{x}$ to the i-th nearest neighbor at position $\vec{x}+\Delta\vec{x}_i$ is given by

$$f_i(\vec{x}+\Delta\vec{x}_i, t+\Delta t) - f_i(\vec{x},t) = \Omega_i(\vec{x},t) \tag{1}$$

where Ω_i is an operator which describes the collisions of particles in a statistical way. If this operator is set to zero the left hand side of the above equation describes fluid particles which travel without any interaction from one lattice site to another. The collision operator is simplified using the Bhatnagar-Gross-Krook (BGK) approximation [6]

$$\Omega_i(\vec{x},t) = \frac{1}{\tau}\left(\overline{f}_i(\vec{x},t) - f_i(\vec{x},t)\right) \tag{2}$$

with $\overline{f}_i(\vec{x},t)$ being the equilibrium distribution function and τ the relaxation time. The macroscopic quantities density ρ and velocity $\vec{u}$ are defined as

$$\begin{aligned} \rho &= \sum_i f_i \\ \rho\vec{u} &= \sum_i \vec{v}_i f_i \end{aligned}$$

To obtain conservation of mass and momentum the equilibrium is required to satisfy

$$\rho = \sum_i \overline{f}_i$$
$$\rho\vec{u} = \sum_i \vec{v}_i \overline{f}_i$$

Then the equilibrium function can be derived using the Chapman-Enskog expansion

$$\overline{f}_i = \rho\left(a + b\vec{v}_i \cdot \vec{u} + c(\vec{v}_i \cdot \vec{v})^2 + d|\vec{u}|^2\right)$$

with a, b, c, d being constants dependent on the lattice geometry. Eq. (1) can be shown to recover the full non-linear Navier-Stokes equations for small local deviations from the equilibrium [3]. For the particular lattice used here the equilibrium distribution function reads [1]

$$\overline{f}_i = \rho w_i \left(1 + 3\vec{v}_i \cdot \vec{u} + s\frac{9}{2}(\vec{v}_i \cdot \vec{u})^2 + \frac{3}{2}|\vec{u}|^2\right)$$

with $w_0 = 4/9$, $w_1 = w_3 = w_5 = w_7 = 1/9$ and $w_2 = w_4 = w_6 = w_8 = 1/36$. The speed of sound in the fluid at rest is

$$c_s = \frac{1}{\sqrt{3}} \frac{\Delta x}{\Delta t}$$

and the kinematic viscosity is connected to the relaxation parameter via

$$\nu = \frac{2\tau - 1}{6}$$

Here Δx refers to the shortest distance between two nodes.

3 Results

In a first series of test calculations the propagation of a 2D-sound wave in a fluid which is otherwise in uniform motion was simulated. Here a benchmark problem was chosen similar to a problem posed for the second computational aeroacoustics workshop on benchmark problems, Tallahassee, Florida, November, 1996. This problem consists of an initial pressure distribution with a Gaussian shape centered at $x = 0$ that is released in a constant uniform flow with a velocity of Mach = 0.5. Though the Mach number usually would be too high to apply a Lattice-Boltzmann scheme as described above, in this case the simulation performed well up to the desired time step. Soon after the results given below were observed, the errors due to an instability grew larger than the amplitude of the sound waves. Fig. 2 shows the pressure as a function of position after 100 time steps of the model. The effect of the convection of the sound wave is clearly visible, the whole structure is shifted towards higher values of x where the "ground speed" of the sound wave traveling upstream is one third of the wave traveling downstream. The relative deviation from the analytical result is below 1.5%, which clearly shows the ability of the model to simulate the convection effect on the sound waves.

For the second set of test calculations a two dimensional geometry was defined which is similar to usual ultrasound gas flow meters. The gas flow meter is made from a U-shaped pipe where the test section is in the lower horizontal part (see fig. 3). A sound wave that enters the test section will be convected by the mean flow which results in an altered transit time to the microphone at the opposite side of the test section. If a sinusoidal sound signal is used this can be measured using correlation techniques as an altered phase difference between the signal from the source and the microphone. The boundary conditions at the inlet were normal density and at the outlet uniform flow with constant velocity. The test signal was a 40 kHz sinusoidal sound wave with an amplitude of 0.2 Pa. Further parameters are given in table 1.

Fig. 3 also shows a snapshot of the pressure field in absence of a mean flow. The sound wave can be seen to propagate through the test section and also enter the inlet and outlet. Thus a complicated pattern of standing waves is generated soon after the simulation is started.

Fig. 4a shows a snapshot of the absolute value of the velocity of the fluid with a mean flow of 5 m/s and the sound source turned on. The flow enters the setup being laminar, but becomes turbulent when it enters the test section. The generation of eddies can be seen at the discontinuities near the sound source and the

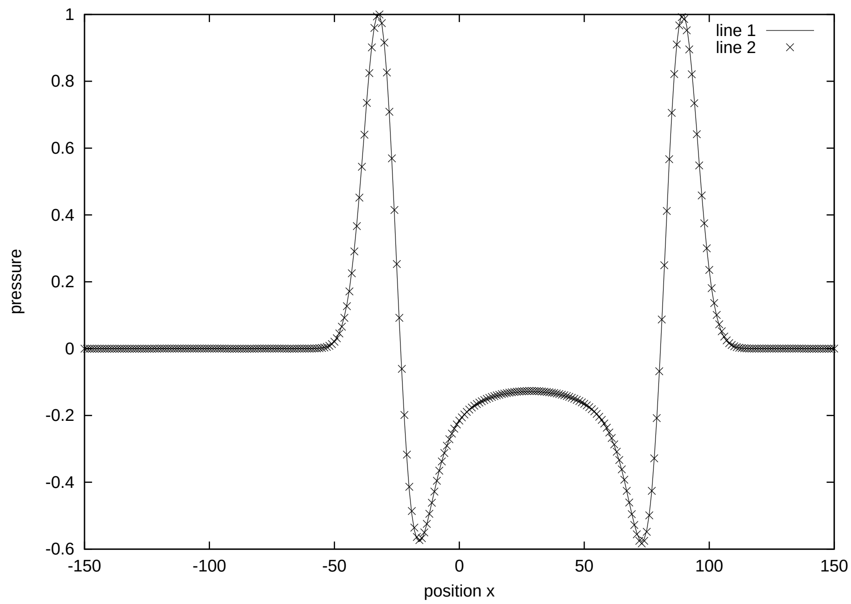

Figure 2: Pressure as function of the position 100 time steps after the release of an initial pressure distribution with Gaussian shape, released at $x = 0$. The uniform background flow has a Mach number of 0.5. The pressure distribution is shifted to larger values values of x which is due to the convection effect on the sound waves. The solid line gives the analytical solution, the numerical result is indicated with x-Symbols.

number of nodes	102629
diameter of the pipes	0,012 m
flow velocity	1 ... 10 m/s
Reynolds number (referred to diameter)	800 ... 8000
density	1,21 kg/m^3
speed of sound (fluid at rest)	340 m/s
kin. viscosity	1,5x10^{-5} m^2/s
spatial resolution	2x10^{-4} m
time step	3,4x10^{-7} s

Table 1: Parameters of the test calculation for the ultrasound gas flow meter

microphone. The velocity variations due to the sound wave are much smaller than the hydrodynamic effects and therefore cannot be seen on the image, although the 40 kHz pressure wave is well detectable in the microphone signal. In a further setup, a fixed cylinder was placed in the inlet which generated additional turbulence (see fig. 4b) . This was to study the influence of flow field modifications on sound propagation.

To compute the phase difference of the signal generated at the sound source the pressure was recorded as a time series at the position of the microphone.

The numerical experiments were conducted for both setups with mean velocities ranging from 1 m/s to 10 m/s in steps of 1 m/s. Each simulation ran for 300000 to 500000 time steps, which corresponds to a time interval of 0.1 to 0.17 s.

In all cases the flow was turbulent, where in absence of the additional turbulence generator the flow became significantly instationary not before the test section.

Fig. 5 depicts the time averaged phase difference between the source and the microphone signal and its standard deviation as a function of the mean velocity, respectively. Up to about 8 m/s the time averaged phase difference is linearly dependent on the mean velocity. At the same time the variability grows with increasing velocity, which is due to stronger generation of turbulence. At velocities higher than 8 m/s the variability of the phase difference is so large that no time averages can be calculated. The statement also holds if a cylinder is put into the inlet which produces additional turbulence. However, the rate at which the time averaged phase difference increases with velocity is raised about 10% by the cylinder, which means that the gas flow meter would deliver different results. A visual inspection of the magnitude of the flow velocity shows that the flow seems to form a faster jet with a smaller diameter when a turbulence generating cylinder

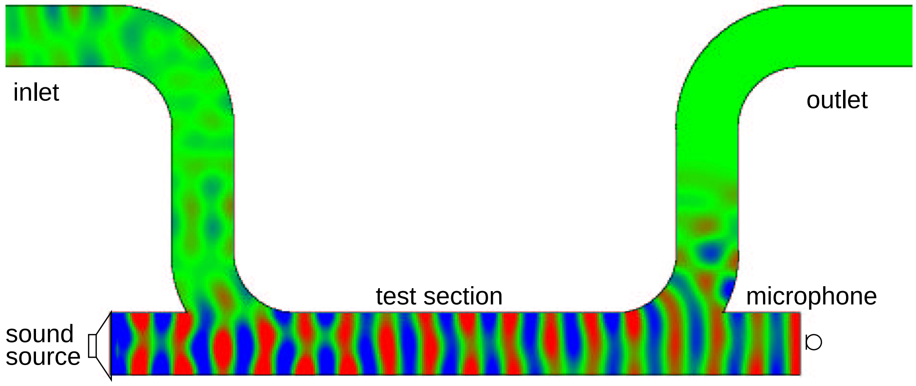

Figure 3: Geometry of the ultrasound gas flow meter. The flow enters at the upper left end and exits at the upper right end. The sound source is situated at the lower left end of the test section while the microphone is on the lower right. The colors indicates the pressure at an early stage of the simulation while the mean flow is set to zero. The red and blue areas correspond to pressure maxima and minima, respectively.

is present in the inlet. This increased velocity compared to the ”laminar” case might be responsible for the observed effect on the phase difference measurements.

4 Conclusions

A 2D simulation of convection effects on sound waves was performed using a Lattice-Boltzmann scheme. Applied to a benchmark problem the results achieved with the Lattice-Boltzmann code showed excellent agreement with the analytic solution available for that case. As a further test an ultrasound gas flow meter geometry was used to perform a flow simulation along with an evaluation of the phase difference between a source and a microphone signal, which serves as a measure for the mean fluid velocity. The time averaged phase difference shows a linear dependency on the mean velocity. By introduction of an additional turbulence generator in the inlet of the flow meter, an alteration of proportionality factor between the time averaged phase difference and the mean velocity was achieved. Although this result has to be confirmed by experiments in future, it indicates that the method is potentially sensitive enough to predict the effects of different inflow condition on the measurements of ultrasound gas flow meters.

Acknowledgements
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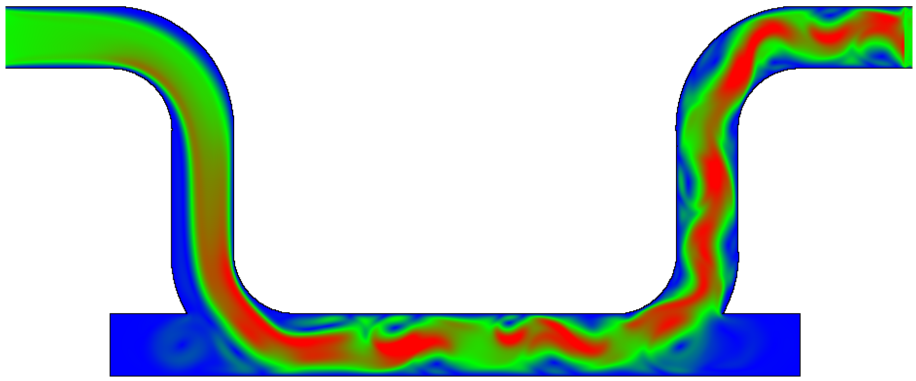

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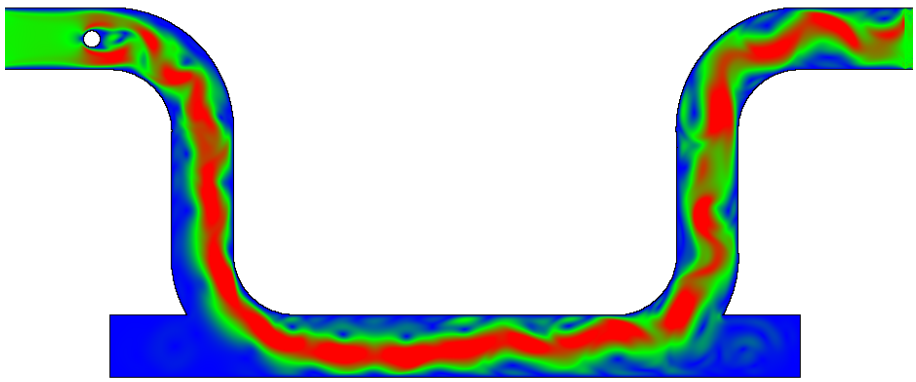

Figure 4: a: Absolute value of the fluid velocity after 300000 time steps. Blue color indicates fluid at rest while red means 10 m/s. The mean flow velocity was 5 m/s. b: Same as a, but with cylinder generating turbulence in the inlet.

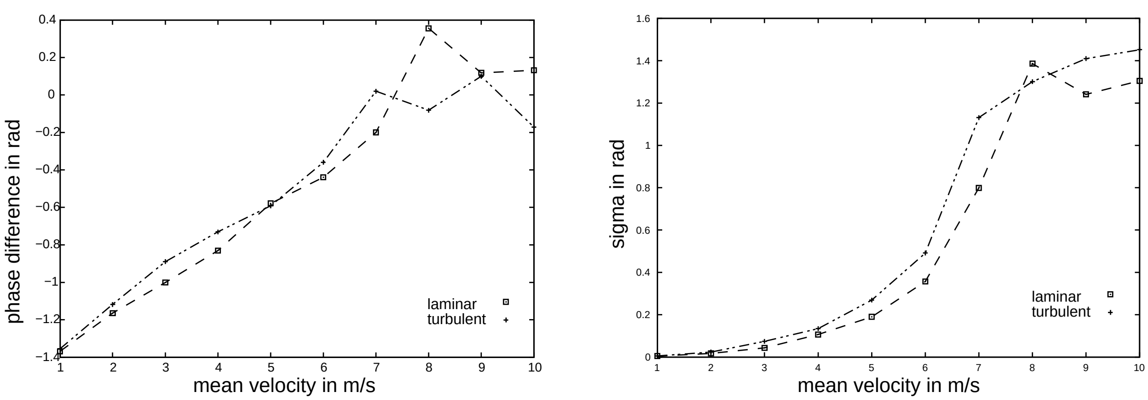

Figure 5: Left: Phase difference of the source and the microphone signal as a function of mean velocity. Up to 8 m/s the phase difference is approximately linear dependent on the mean velocity. Right: At higher mean velocities the standard deviation of the phase difference σ becomes too large to evaluate the phase difference correctly. The label ”turbulent” and ”laminar” refer to the inlet conditions, i.e. with or without turbulence generator in the inlet.