TIME-DOMAIN SIMULATION OF REED WOODWINDS WITH APPLICATION TO MUSICAL SOUND SYNTHESIS

PACS REFERENCE : 43.75.Ef

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ABSTRACT

Various methods for time-domain simulation of woodwind instruments have been developed over the past few of decades. Part of these have been principally aimed at understanding the mechanisms of sound generation in woodwinds, while others have been developed mainly for application to musical sound synthesis. This paper reviews techniques for linear modelling of the bore in the context of musical sound synthesis, and discusses a new method that overcomes some of the problems of previously developed methods.

INTRODUCTION

This paper discusses simulation of reed woodwind tones via physical modelling, using a time-domain (TD) formulation. The advantages of using physical modelling in comparison with other sound synthesis methods have been discussed in detail in various previous studies (see for example [1]). For physical modelling of wind instruments, the bore is usually assumed to behave linearly (except at high amplitude oscillations), and the generation of self-sustained oscillations in the bore is simulated with a model of the interaction between the reed and the bore.

For application in a musical context, the final physical model that is used for the simulation of woodwind tones should fulfil a certain set of criteria. Jaffe [1] has discussed a large set of criteria for sound synthesis methods in general; for the specific case of simulation of the bore of a reed woodwind instrument, we can identify three main criteria: accuracy, efficiency, and tonehole parameterisation. The first of these is obvious; in order to generate realistic sounds, the laws and equations embedded in the model should form an accurate representation of the physical mechanisms of the real instrument. The second criterion is typically associated with application in a musical context, in which it is often considered desirable to run and control the simulation in real-time. Due to hardware limitations such as processing power, a certain level of efficiency is necessary to enable real-time simulation. Even for simulations that are not running in real-time, the effectiveness of the exploration of the musical potential of a physical model largely depends on the trade-off between accuracy and computational efficiency. The third criterion stems from the fundamental musical concept of pitch variation. In a real woodwind instrument, the player controls the pitch via opening and closing toneholes. For simulation of note-to-note transients, a physical model should be constructed in such a way that the state of each tonehole can be adjusted continuously. Moreover, such dynamic control of the tonehole states should be simple and efficient, preferably using only a small number of arithmetic operations.

In this paper, previously developed methods for modelling the bore are reviewed in the light of these criteria, and a new method (the Wave Digital Modelling Method) is briefly discussed and applied to the simulation of clarinet tones.

REVIEW OF METHODS FOR LINEAR TIME-DOMAIN MODELLING OF THE BORE

Several different techniques for linear TD modelling of wave propagation in an acoustic bore can be found in the literature. Under the assumption of linearity, wave propagation is governed by a three-dimensional wave equation. TD simulation can thus be considered as solving a wave equation for certain given boundary conditions and input-signals. Note that for woodwind bores, the boundary conditions are not necessarily constant; for example, opening a tonehole amounts to changing a boundary condition. Standard techniques for numerically solving a wave equation can be applied; perhaps the best known of these are Finite-Difference (FD) methods and Finite-Element (FE) methods, which both are highly accurate and thoroughly tested techniques. However, even for today's standards of commonly available processing power, such numerical formulations are computationally far too expensive for direct application to musical sound synthesis.

More efficient descriptions can be obtained by making two simplifying assumptions. First, the main bore of a musical woodwind instrument is usually either cylindrical or conical, or approximately one of these. As a consequence, only the primary mode can propagate at frequencies below cut-off throughout the instrument, i.e., all higher modes are evanescent at frequencies which are of interest to the sound generation mechanism. Wave propagation in the cylindrical and conical parts of the bore may therefore be described with a one-dimensional wave equation. Small units such as toneholes can be modelled as lumped elements, while the bore termination may also be treated as a lumped element. Second, the acoustic variables do not have to be known at each position inside the air column. For example, for modelling the interaction between the bore and the driver, only the pressure and volume velocity at the mouthpiece-end need to be computed. In principle, the bore itself may be characterised by an impulse response; for a given volume velocity, the mouthpiece pressure can be computed by means of convolution. A general framework for simulation of sustained musical tones on this basis was developed by McIntyre et al. [2], and over the last few decades, several specific woodwind instrument applications of this convolution (CV) approach have been developed (see for example [3]). The efficiency of CV methods largely depends on the length of the impulse response.

Alternatively, the bore response can also be modelled as a finite set of resonances. Typically, only the first 5 to 10 resonances influence the sound generation mechanism, thus the response of the bore may be represented with just a small number of natural modes, each modelled as a second-order oscillator. Such an approach, which is often referred to as modal synthesis (MS), has been taken by Adrien [4], and has recently been proposed for simulation of brass instruments by Vergez [5]. Because of the truncation of the number of modes, MS is generally less accurate but more efficient than CV methods.

The main disadvantage of both CV methods and MS is that they do not allow for a direct form of tonehole parameterisation. With CV methods, the impulse response only represents one particular configuration of the tonehole states; for dynamic modelling of the toneholes, a large set of different impulse responses would have to be calculated off-line, and the pressure response for time-varying tonehole states could in principle be computed by interpolating between these impulse responses. A similar formulation could be used in the MS approach. However, such parameterisation is difficult and elaborate, and adds significantly to the overall computational load.

A much more efficient and simple parameterisation is possible with a one-dimensional wave propagation model in which the acoustic variables are computed at all the tonehole positions. This way, the effect that the opening and closing of a tonehole has on the bore response can be modelled on basis of the relatively simple mathematical relationship between the local acoustic variables and the tonehole state. Further efficiency is gained by modelling the acoustic variables at all discontinuities in the bore. That is, the acoustic variables are computed at all points in the bore at which a one-dimensional wave is reflected; we will refer to this approach as travelling-wave (TW) methods. Table 1 summarises the properties of the different methods for linear modelling of the bore.

METHOD	ACCURACY	EFFICIENCY	TH PARAMETERISATION
FD	*high*	*low*	*yes*
FE	*high*	*low*	*yes*
CV	*high*	*medium*	*no*
MS	*medium*	*high*	*no*
TW	*medium*	*high*	*yes*

Table 1: Properties of various methods for linear TD modelling of the bore.

Two methods that are based on the concept of transmission and reflection of travelling waves can be found in the literature, namely digital waveguide modelling (DWM) and the multi convolution algorithm (MCA). These approaches differ mainly in the details of the numerical formulation. The main advantage of the DWM approach is that it allows the adjustment of the balance between accuracy and efficiency. Frequency-dependent phenomena (such as boundary or radiation losses) are modelled using a digital approximation of a continuous-domain formulation, where both the continuous formulation and the digital approximation technique may be chosen freely. In this respect the MCA approach is more limited, since it relies on the possibility of performing analytic inverse Fourier transforms of continuous-domain formulations.

For both methods, instability problems can occur in numerical simulations of conical sections. While it has been shown that (1) continuous-time TW models of conical bore systems are in principle stable [6,7,8], and (2) both the DWM and the MCA approach can be used to compute the discrete impulse response of an acoustic bore that contains conical sections without any instability problems [9,10], it has yet to be shown under which conditions either method actually remains stable. In fact, Scavone [9] has reported numerical instability when using DWM techniques with long simulation times.

Another obstacle of the DWM approach is that it does not provide methods for modelling of toneholes in a conical bore. This shortcoming stems directly from the fact that DWM techniques are specifically defined to simulate distributed systems; as a consequence, incomputable loops are created when a conical section is directly connected to an acoustic unit with a non-zero instantaneous reflection (such as a tonehole). In the MCA approach, this particular problem is avoided by lumping the tonehole and the cone taper discontinuities together.

THE WAVE DIGITAL MODELLING METHOD

The wave digital modelling method (WDM) makes strong use of the classical analogy between electrical and acoustical systems, and combines DWM techniques with wave digital filter (WDF) techniques. WDF techniques are used for discretisation of analog networks [11]. The resulting digital networks are called *wave digital filters* (WDFs). The classical analogy between electric and acoustic systems raises the possibility of employing WDF techniques for the discretisation of lumped elements in a model of an acoustic system. WDF techniques are similar to DWM techniques in the sense that they both digitise continuous-time models using *wave variables*. As already suggested by Smith [12] and recently elaborated by Bilbao [13], a combined approach is possible. In the present study, lumped elements are modelled using WDF techniques and distributed elements are modelled using DWM techniques.

The procedure for the derivation of the wave digital model of an individual bore component is similar to the derivation of a wave digital filter, and consists of three steps:

(1) decomposition of the acoustic variables into wave variables.
(2) discretisation of frequency-dependent elements.
(3) satisfaction of the computability condition.

Step (1) is accomplished by using the following relationships:

$$p_i = p_i^+ + p_i^- \quad , \quad R_i U_i = p_i^+ - p_i^- \tag{1}$$

where for port *i*, p_i is the pressure and U_i is the volume velocity, while p_i^+ and p_i^- are the wave variables. The quantity R_i has the dimension resistance and, following WDF theory, is referred to as the *port-resistance*. In the case of a distributed acoustic element, the wave variables represent pressure-waves travelling through a certain medium. The port-resistance then equals the reference impedance that characterizes the medium; in the case of a wave travelling through an air-filled pipe, this is the characteristic impedance $Z_0 = \rho c / S$, where *S* is the cross-sectional pipe area, ρ is the mean air density, and *c* is the wave velocity. In the case of a lumped acoustic element, the wave variables do not represent waves that actually travel any distance; the decomposition is in this case merely a matter of mathematical description, and from an acoustical point of view the port-resistance may then be considered arbitrary. As in the derivation of WDFs, this freedom of choice is exploited to avoid delay-free loops in the final modelling structure.

Step (2) concerns the approximation in the digital domain of linear, frequency-dependent, continuous domain phenomena, which is realized in the present study by means of digital filter techniques. Lumped circuit elements, such as inertances and compliances, are discretised via the bilinear transform. Discretisation of other frequency-dependent phenomena, such as viscothermal losses, are predominantly carried out by means of infinite impulse response (IIR) filters, while Lagrange FIR interpolation filters [14] are employed for digital approximation of fractional delay lengths.

Step (3) is concerned with the computability of the resulting digital structure. Like a digital filter, a wave digital model is described mathematically by a system of difference equations. Such a system is called *computable* if the arithmetic operations prescribed by these equations can be ordered sequentially at each discrete-time instant [11]. In practice this condition is satisfied if the system contains no delay-free loops. In a wave digital model, such delay-free loops may arise in the discretisation of a lumped element. Following WDF theory, these loops are ensured to have at least one delay by choosing the appropriate port-resistance of that loop.

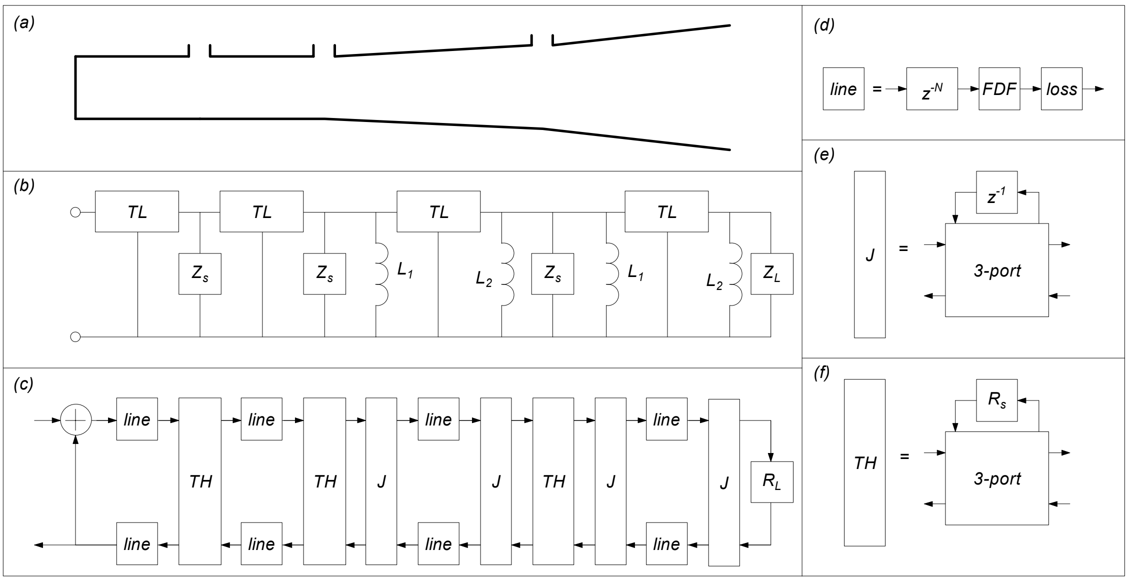

Figure 1: Wave digital modelling of a woodwind bore. (a): Example bore. (b): Equivalent network. Each *TL*-unit indicates a uniform transmission-line, and Z_s is the open-end impedance. (c) Wave digital model. R_s is the open-end reflectance. (d): Discrete-time model of lossy wave propagation in a conical or cylindrical section using a cascade of a delay-line, a fractional delay filter *(FDF)*, and a loss-filter. (e): A conical junction inertance modelled in discrete-time as a single delay, interfaced to the main bore with a three-port junction. (f): Wave digital tonehole model, using a wave digital reflectance filter *(R_s)*, which has coefficients that depend on a single tonehole state parameter.

SIMULATION OF PIECEWISE CONICAL BORES WITH TONEHOLES

A woodwind bore may be considered as a succession of conical and cylindrical sections with a set of open or closed holes in their sides [15]. As pointed out by Benade [16], a conical section may be described with "an equivalent circuit consisting of a pair of inertances, a transformer, and a non-tapered duct that has the same length and small-end radius as the cone to be represented". A tonehole may be described as a shunt impedance [17]. Using these descriptions, a piecewise conical bore with toneholes may thus be modelled as depicted in figure 1. A junction without a tonehole can be modelled as a single inertance defined as the parallel combination of L_1 and L_2. The transformers are omitted from the network description, which does not affect the response of the bore at the bore entry. For each conical section, the junction inertances at the entry and the end are defined as

$$L_1 = \rho r_1/S_1 \quad , \quad L_2 = -\rho r_2/S_2 \ , \qquad (2)$$

where r_1, S_1 and r_2, S_2 are the distance to the cone apex and the cross section at the left and the right end of the cone, respectively. For a partially open tonehole, the shunt impedance may be described as an inertance in parallel with a compliance [18]. The wave digital tonehole model (see Figure 1f) , that is directly derived from this formulation, takes the form of a three-port junction with a wave digital reflectance filter, the coefficients of which depend on a single control parameter that varies between 0 (=closed) and 1 (= open) .

A discrete-time model can be derived by applying the steps described in section to each of the components, which results into the discrete-time structure depicted in Figures 1d, 1e, and 1f; the complete discrete-time modelling structure of an example woodwind bore is shown in Figure 1c. For details of the derivation of these structures, we refer to [18,19].

With regard to the stability of conical bore simulations, the method was tested for various bore configurations [18]; the main conclusion is that the simulations are stable only in cases where viscothermal losses are not taken into account in any of the conical sections. Furthermore, the results indicated that for stable simulation of lossy conical sections, a consistent formulation of the propagation constant has to be employed. That is, the junctions have to formulated with the same propagation constant as the transmission-lines; this is not the case for the WDM method when using equations (2), and also not for other TW methods available in the literature.

CLARINET SIMULATION

The dimensions of the main bore and the toneholes of a Selmer clarinet (no. 1400) were measured. The part of the bore starting directly after the hole that is closest to the open end is considered as the bell. The bell is typically mildly flared, and may be approximated with a small number of piecewise conical sections. The inter-hole sections are assumed to be approximately cylindrical; calculations with a transmission-line model showed that this simplification causes only a very small error. The mouthpiece is modelled with a tapered section (which connects to the main bore) and a cylindrical section (that represents the entry section to which the reed is clamped). Figure 2 shows the input impedance of the complete clarinet bore as computed with the transmission-line model and the wave digital model. Note that the differences between the two impedance curves are extremely small.

For generation of sustained notes, this wave digital model of the clarinet bore has to be coupled to a reed excitation model. For this purpose, the non-linear reed oscillator described in [18,20] is a suitable candidate, since it takes into account the effects of the reed curling on the mouthpiece lay. The parameters of this lumped reed model are varying with reed deflection, and are derived from a finite-difference simulation of the reed-mouthpiece-lip system. As shown in [18], this phenomenon has strong influence on the timbre of the radiated sound.

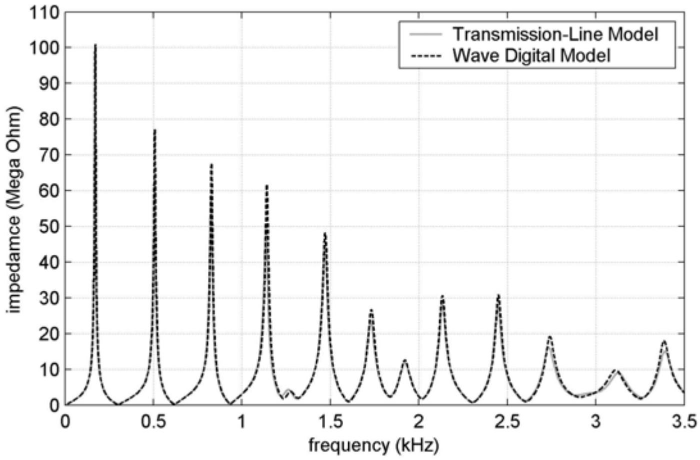

Figure 2: Input impedance of the Selmer clarinet, with fingering for note F_3.

CONCLUSIONS AND FUTURE RESEARCH

Simulation of woodwind tones was discussed, with emphasis on linear modelling of the bore. The wave digital modelling method was discussed, and applied to the simulation of a Selmer clarinet. Comparison with a transmission-line model show a high accuracy at frequencies below cut-off. Given that the WDM method is efficient, and allows direct tonehole parameterisation, it forms a suitable basis for physical modelling of woodwinds. However, future research is required for stable simulation of conical bore systems with inclusion of viscothermal losses.

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