Unified modelling of simplified musical instruments in time domain

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Abstract: Musical instruments contain, at their very basis, a vibrating element (string or air column) which can be treated as a 1D linear element. Its oscillation is obtained either through a punctual excitation (or initial condition) or by means of a continuous energy input through a non linear device. We will show that it is possible to establish a general iterative formulation containing delayed variables which can be adapted to each particular case (cylindrical or conical woodwind like systems, plucked or bowed strings).

I) INTRODUCTION

Physical modelling can be used for two different purposes, that of sound synthesis following as much as possible the sound of real instruments sound, and that of physical understanding of sound production in musical instruments. These two goals induced during the past twenty years a number of scientific works following the most outstanding one of McIntyre et al. [McIntyre, 1983]. McIntyre et al. present a very simple model for self sustained instruments based on two equations, a convolution containing the whole time history of the impulse response of the resonant system (mainly air columns and strings) and a non linear equation associated with the excitation device. Of course, this model can also be used to study instruments based on free oscillations if the non linear equation is substituted by a punctual energy supply. Despite its simplicity, this model has proven its efficiency for the physical understanding of musical instruments (see for example the works of Maganza et al [Maganza, 1986], Grand [Grand, 1994] or Barjau et al. [Barjau , 1997] among others).

The aim of the present work is to go a step further and generate new sounds (corresponding to virtual instruments) from this same kind of model by just changing the non linear excitation function. The resulting models will be “semi physical models” as they keep the physical description of the resonant part while choosing freely the non linearity (with the only constraint of giving stable oscillations).

We will first describe a generic model for self-sustained instruments that can also be used for plucked or strucked instruments in the absence of non linearity. This model is intimately related with those presented in Barjau et al. [Barjau, 1997] where a woodwind is modelled through differential equations with delays.

We will then particularise this description for stringed instruments, and use it with different non linear bowing functions not necessarily realistic. The numerical implementation of the models will show that a variety of functions lead to interesting sounds from the musical point of view.

II) GENERIC MODELS

The generic models we will use follow the same time-domain scheme as those presented in the pioneering work of Schumacher [Schumacher, 1981] and MacIntyre et al. [Mc Intyre, 1983]. Time domain is more suitable than frequency domain for music not only because music is essentially the time occurrence and organisation of sounds but also because the non linearity (essential in self sustained behaviours) is better dealt with in such domain.

A general scheme for a 1D general oscillating-musical system could be the following. The main resonant part (strings or air columns) is represented through an idealised 1D linear system having two absorbing-reflecting ends whose role is that of energy transfer (either to a resonant secondary system or directly to the surrounding air).

The 1D system is set into oscillation when an amount of energy is fed into it by a continuous or a discrete system (the excitation device) that can be linked to the resonant system in a linear or a non linear way. As a first approach, the excitation device can be localised at a single point.

If just uniform strings are considered, their dynamical description can be reduced to a propagation speed and two characteristic time constants corresponding to the time intervals needed for a round trip along the two string spans defined by the excitation point and the two ends. If the excitation point is located at one end, only one time constant is needed. This is the case of many wind instruments whenever their air column can be considered as a uniform one.

The whole problem can be seen as a forced one or as a coupled problem. In the first case, the dynamical behaviour of the excitation device is neglected and its response is considered as instantaneous. An algebraic equation is sufficient to represent mathematically its action on the main resonant system.

In the second case, the excitation device can be modelled as a resonant system having also two absorbing-reflecting ends, a propagation speed and two characteristic time constants. The excitation point can be represented as a transmission-reflection point coupling the two resonant systems.

If the main resonant part is modelled as a 2D or 3D system but its coupling with the excitation device is still localised at one single point, it is not possible to talk about two characteristic time intervals any more due to the infinite (and uncountable) number of "characteristic lengths" associated with the infinite number of straight paths going from the excitation point to the boundaries of the system.

III) THE ALGEBRAIC MODEL

The simplest model corresponding to the forced problem presented in the previous section is the well-known and efficient one used by Maganza et al. [Maganza, 1986] to mimic a clarinet. A simplified description of the resonant system through an impulse response containing a set of Dirac impulses and a suitable change of variables lead to a model where the Non Linear function is iterated at a period $T = l / c$ (where l is the bore length). The generic equation associated to this model is then $X(t) = F_{NL}(X(t-T))$.

A similar model can be used for a bowed string. The starting point is the convolution integral relating the transversal velocity of the string at the excitation point $v(t)$ and the friction force of the bow $f(t)$ through the impulse response of the string $h(t)$: $v(t) = h(t) * f(t)$.

It is useful to split the impulse response of a string into a short time response $h_0(t)$ and a long time one $h^*(t)$: $h(t) = h_0(t) + h^*(t)$. As the excitation point is placed somewhere between the two string ends, the short time response has half the intensity of that which would be obtained if excited at one end. If we call $h_0(t)$ the short time response at one end, then the short time response for the intermediate point will be $\frac{1}{2} h_0(t)$.

The long time response $h^*(t)$ is obtained from a propagation-reflection rationale of the initial upward and downward waves. It is useful to decompose $h^*(t)$ into two parts, $h^*(t) = h_1^*(t) + h_2^*(t)$, each of them containing the propagation-reflection process of one of these two initial waves.

If we call $R_1(t)$ and $R_2(t)$ the elementary reflection functions associated with the two ends, $\boldsymbol{s}_1(t)$ and $\boldsymbol{s}_2(t)$ the damping functions associated with the propagation along the two string lengths (from bow to bridge and from bow to knut) $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ the two characteristic times, the long time impulse responses $h_1^*(t)$ and $h_2^*(t)$ are:

$$
\begin{aligned}
h_i^*(t) = {} & \sum_{n=0}^{\infty} \left[R_i(t) * \boldsymbol{s}_i(t) *\right]^{n+1} \left[R_j(t) * \boldsymbol{s}_j(t) *\right]^n \frac{1}{2} h_0(t - \boldsymbol{t}_i - n\boldsymbol{t}) + \\
& + \sum_{n=1}^{\infty} \left[R_i(t) * \boldsymbol{s}_i(t) *\right]^n \left[R_j(t) * \boldsymbol{s}_j(t) *\right]^n \frac{1}{2} h_0(t - n\boldsymbol{t})
\end{aligned}
,
$$

where *i=1, j=2* for $h_1^*(t)$ and *i=2, j=1* for $h_2^*(t)$, and $\boldsymbol{t} = \boldsymbol{t}_1 + \boldsymbol{t}_2$. The substitution of these expressions into the convolution integral leads to:

$$
\begin{aligned}
v(t) = {} & \frac{1}{2} v_0(t) + \sum_{n=1}^{\infty} \left[R_1(t) * \boldsymbol{s}_1(t) *\right]^n \left[R_2(t) * \boldsymbol{s}_2(t) *\right]^n v_0(t - n\boldsymbol{t}) + \\
& + \frac{1}{2} \sum_{n=0}^{\infty} \left[R_1(t) * \boldsymbol{s}_1(t) *\right]^n \left[R_2(t) * \boldsymbol{s}_2(t) *\right]^n R_1(t) * \boldsymbol{s}_1(t) * v_0(t - \boldsymbol{t}_1 - n\boldsymbol{t}) + . \\
& + \frac{1}{2} \sum_{n=0}^{\infty} \left[R_1(t) * \boldsymbol{s}_1(t) *\right]^n \left[R_2(t) * \boldsymbol{s}_2(t) *\right]^n R_2(t) * \boldsymbol{s}_2(t) * v_0(t - \boldsymbol{t}_2 - n\boldsymbol{t})
\end{aligned}
$$

where $v_0(t) = h_0(t) * f(t)$.The summations can be eliminated by combining the last equation with its shifted expression at time $t - \boldsymbol{t}$:

$$
\begin{aligned}
v(t) - \frac{1}{2} v_0(t) = {} & R_1(t) * R_2(t) * \boldsymbol{s}_1(t) * \boldsymbol{s}_2(t) * \left[v(t - \boldsymbol{t}) + \frac{1}{2} v_0(t - \boldsymbol{t}) \right] \\
& + \frac{1}{2} \left[R_1(t) * \boldsymbol{s}_1(t) * v_0(t - \boldsymbol{t}_1) + R_2(t) * \boldsymbol{s}_2(t) * v_0(t - \boldsymbol{t}_2) \right]
\end{aligned}
.
$$

This equation involves just a short part of the time history of $v(t)$ and $v_0(t)$, as McIntyre et al. state in [McIntyre, 1979]. Different choices for $R_i(t)$, $\boldsymbol{s}_i(t)$ and $v_0(t)$ lead to different models.

The algebraic model is obtained when internal dissipation is neglected (that is, $\sigma_i(t) = \delta(t)$) and the reflection functions are taken to be $R_i(t) = -R_i\delta(t)$, with $0 < R_i < 1$. Though the characteristic impedance of the string, $A_0 = 1/\sqrt{T\mu}$ (where T, μ are the string tension and linear mass, respectively), is the proportionality coefficient relating the rightward (leftward) transversal tension with the rightward (leftward) transversal velocity at any point, the relationship between the total initial transversal tension and total transverse velocity at the bowing point is $v_0(t) = \frac{1}{2}A_0 f(t)$ (as the total admittance associated with two coupled systems, each having an admittance A_0, is just $A_0/2$).

The algebraic model is:

$$v(t) - \frac{1}{2}A_0 f(t) = R_1R_2\left[v(t-\tau) + \frac{1}{2}A_0 f(t-\tau)\right] - \frac{1}{2}A_0\left[R_1 f(t-\tau_1) + R_2 f(t-\tau_2)\right] \; .$$

As said before**,** this model is also valid for plucked or strucked strings if one takes $f(t) = I\delta(t)$. In general, $f(t) = F_{NL}[v(t)]$. The resulting free oscillation is presented on figure 1 . It is an algebraic model that can be used for any case of 1 D musical instrument including woodwinds.

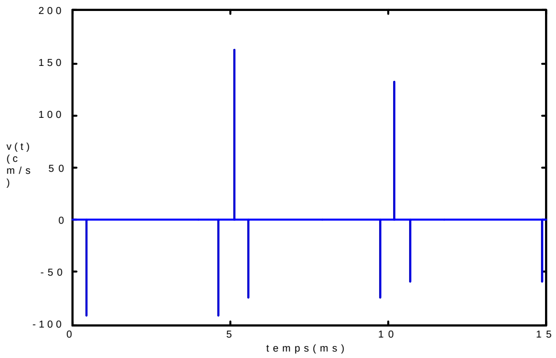


Figure 1 : Free oscillation of the “algebraic” string.

IV) NUMERICAL RESULTS

The solution for each time t of the iterative equation demands to solve the non-linear equation $v(t) - \frac{1}{2}A_0 F_{NL}(v(t)) = K(t-\tau_1, t-\tau_2)$, that can have more than one solution. This equation has been used for computing various waveforms, using Non Linear functions of the type presented on figure 2. This non linear function is the classical one representing the non linear interaction of a bow and a string. It can be used with or without an offset at infinity, in the normal way as presented or in a reversed way replacing F_{NL} by - F_{NL} .

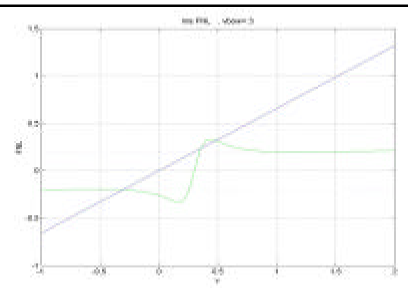
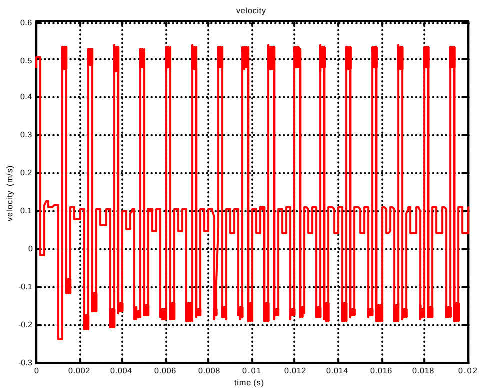
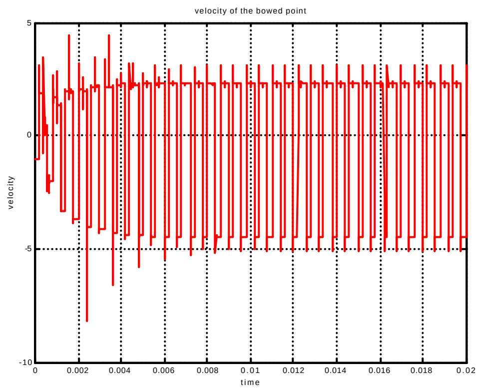


Figure 2 : An example of a classical non linear function

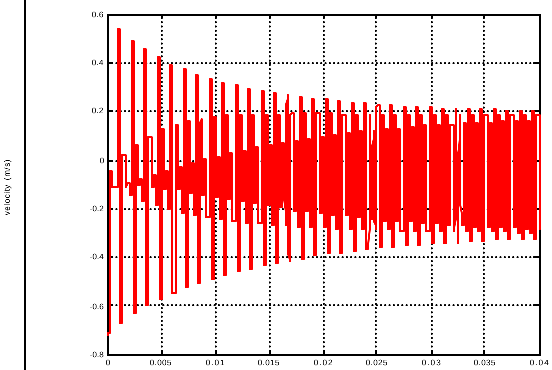
On figure 3 it is possible to compare several waveforms obtained with the algebraic model with two different non linear functions. It is obvious that the effect on the waveform and so on the sound is mainly due to the shape of the nonlinear function. Some simulations present unstable behaviours that are clearly related with the number of intersection points solution of the nonlinear algebraic equation.



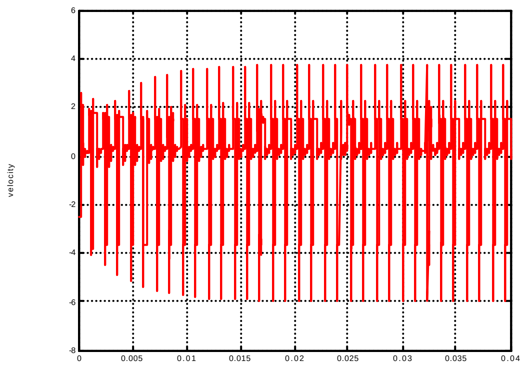
velocity at the bowing point with a "classical" non linearity



velocity at the bowing point with an "reversed" non linearity



velocity at the bowing point with a " classical " non linearity



velocity at the bowing point with an "reversed" non linearity

Figure 3 : Examples of synthetized velocity

VI) CONCLUSION

The algebraic model we have presented, can be used as a generic model for a musical instrument that can be described in a 1D domain. It presents physical behaviours that can be compared to those of a real instrument but it can also be used as a model for virtual instruments only by a changing its non-linear part.

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