SOUND RADIATION FROM VISCO-ELASTICALLY DAMPED PLATES EXCITED BY AN IMPACT FORCE

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ABSTRACT

Transient sound radiation from baffled rectangular plates with free viscoelastic layers excited by an impulsive point force is studied both analytically and experimentally. The contact force developed during impact between a ball and a plate is represented as a point force with a squared half-period sine-wave time history. The vibration responses of the plates are obtained using normal mode analysis, modal parameters being estimated by means of the FEA and the MSE method. The radiated sound wave forms are obtained by numerical integration of the Rayleigh integral. An experimental study is carried out to measure the vibration responses and sound radiation of plates impacted by a steel ball.

INTRODUCTION

Transient sound radiation from elastic structures excited by impact forces is a fundamental problem in industrial noise control and has been extensively investigated with the aim of clarifying the mechanism of sound radiation [1-5]. The use of viscoelastic materials which have been applied to control vibration and noise in automobile, aircraft, railway vehicle structures and electronic devices can successfully decrease impulsive sound radiation from mechanical systems, as damping is effective in controlling the resonant response of elastic structures. Only a few studies, however, have discussed impact sound radiation from elastic structures with viscoelastic layers. Jeon[6] studied the transient sound radiation from a composite, clamped circular plate with viscoelastic layers. The contribution of damping materials to impact sound radiation from a simply supported rectangular plate was studied by Traccaz[7].

In this paper, the transient sound radiation from a clamped rectangular plate with free viscoelastic layer is studied. The contact force is, addressed to the article of Akay[8], represented as a point force with a squared half sine wave time history. The sound pressure waveforms and Fourie spectra are obtained by numerical integration of the Rayleigh integral. These results are compared with corresponding experimental results.

ANALYTICAL METHOD

Calculation of the Plate Response

The vibration response of a thin plate as illustrated in Fig. 1 to an impact force applied at a point (x_0, y_0) can be found by solving the equation of motion of the plate as

$$D\nabla^4 w(x,y,t) + C\frac{\partial w(x,y,t)}{\partial t} + m_p \frac{\partial^2 w(x,y,t)}{\partial t^2} = F(t)\delta(x-x_0)\delta(y-y_0) \qquad (1)$$

where $\delta(\ )$ is the Dirac delta function, $w(x,y,t)$ is the displacement of the plate and $F(t)$ is the applied force. D and m_p are the flexible rigidity and mass per unit area of the plate. The effect of damping of the plate is represented by introducing the damping coefficient C. The fluid surrounding the plate is homogeneous, with a speed of sound c and density ρ much less than the density of the plate and the back reaction of the radiated acoustic pressure is neglected. The displacement response of a plate to an arbitrary force can be written on the assumption of proportional viscous damping as

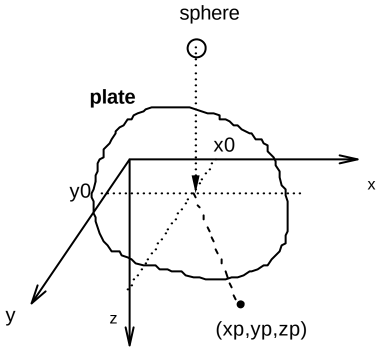

FIG. 1. Coordinate systems.

$$w(x,y,t)=\sum_r \frac{\phi_r(x_0,y_0)\phi_r(x,y)}{m_r\omega_r^*}\int_0^t F(\tau)e^{-\xi_r\omega_r(t-\tau)}\sin\omega_r^*(t-\tau)\,d\tau \qquad (2)$$

$$\omega_r^*=\omega_r\sqrt{1-\xi_r^2} \qquad \xi_r=\eta_r/2$$

where m_r, ω_r, η_r and $\phi_r(x,y)$ are the modal masses, natural frequencies, modal loss factors and mode shapes respectively, with the subscript *r* denoting the mode number.

A method shown by Akay[8] based on the Hertz law of contact is applied to the analytical procedure for calculating the impact force, which is generated by a collision of a sphere with a flat body. The expression for the contact force developed during elastic impact of a sphere of radius R with a rigid plane surface is given as a function of the relative approach α of the sphere and the impacted plane surface:

$$F(t)=k\alpha(t)^{3/2} \qquad k=\frac{4}{3}\sqrt{R}\left(\frac{1-\nu_s^2}{E_s}+\frac{1-\nu^2}{E}\right)^{-1} \qquad (3)$$

where E_s, E and ν_s, ν are Young's moduli and Poisson's ratios of the sphere and impacted body respectively. In the case of impact of a sphere with a large thin plate, the impulsive force-time history is represented as a squared half-period sine-wave time history.

$$F(t)=\begin{cases}F_0\sin^2\omega_0 t & 0\le t\le T_H\\ 0 & t>T_H\end{cases} \qquad T_H=\pi/\omega_0 \qquad (4)$$

where the duration of contact T_H is given by $T_H=2.9432\,\alpha_m/V_0$. $\alpha_m=\left(5V_0^2 m_s/4k\right)^{2/5}$ is the maximum value of the relative approach α, and V_0 is the impact velocity. Following the analysis given by Zener[9], the maximum impact force F_0 is obtained from Fig. 2 by using the inelasticity parameter $\lambda=3.218\,m_s/T_H Z_p$, where m_s is the mass of the sphere and $Z_p=8\sqrt{Dm_p}$ is the driving point impedance of the plate.

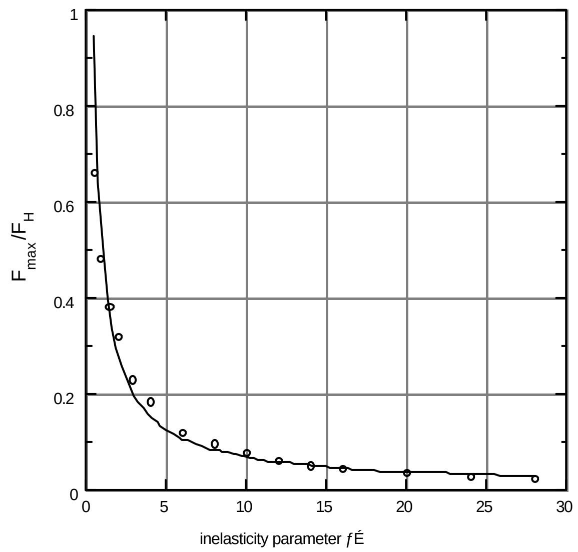

FIG. 2. Impact force amplitude [8].

Substituting Eq. (4) into Eq. (2), the displacement response of a plate to an impact force can be found as

$$w(x,y,t)=\sum_r \frac{\phi_r(x_0,y_0)\phi_r(x,y)}{m_r\omega_r^*}\left[W_1(t)+W_2(t)+W_3(t)\right] \qquad (5)$$

where

(i) for $0\le t\le T_H$

$$W_1(t)=\frac{F_0}{2w_r}\sin\,\Theta_1-\frac{F_0}{2w_r}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_1\right)$$

$$W_2(t)=-\frac{F_0}{4\Omega_2}\sin\left(2w_0 t+\Theta_2\right)+\frac{F_0}{4\Omega_2}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_2\right)$$

$$W_3(t)=\frac{F_0}{4\Omega_3}\sin\left(2w_0 t-\Theta_3\right)+\frac{F_0}{4\Omega_3}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_3\right)$$

(ii) for $t>T_H$

$$W_1(t)=\frac{F_0}{2w_r}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)+\Theta_1\right]-\frac{F_0}{2w_r}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_1\right)$$

$$W_2(t)=-\frac{F_0}{4\Omega_2}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)+\Theta_2\right]+\frac{F_0}{4\Omega_2}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_2\right)$$

$$W_3(t)=-\frac{F_0}{4\Omega_3}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)+\Theta_3\right]+\frac{F_0}{4\Omega_3}e^{-x_r w_r t}\sin\left(w_r^* t+\Theta_3\right)$$

$$\Theta_1=\tan^{-1}\frac{w_r^*}{x_r w_r}=\tan^{-1}\frac{\sqrt{1-x_r^2}}{x_r}\qquad \Theta_2=\tan^{-1}\frac{w_r^*-2w_0}{x_r w_r}\qquad \Theta_3=\tan^{-1}\frac{w_r^*+2w_0}{x_r w_r}$$

$$\Omega_2=\sqrt{x_r^2 w_r^2+\left(w_r^*-2w_0\right)^2}\qquad \Omega_3=\sqrt{x_r^2 w_r^2+\left(w_r^*+2w_0\right)^2}$$

<u>Acoustic Pressure Radiated from a Plate</u>

The acoustic pressure at a point $\left(x_p, y_p, z_p\right)$ radiated from a plate vibrating in an infinite baffle is calculated by Rayleigh integral

$$p\left(x_p, y_p, z_p, t\right)=\frac{r}{2p}\int_S\frac{\ddot{w}\left(x,y,t-d/c\right)}{d}\,\mathrm{d}x\,\mathrm{d}y\qquad d=\sqrt{\left(x-x_p\right)^2+\left(y-y_p\right)^2+z_p^2}\qquad (6)$$

where r is the air density. The plate acceleration $\ddot{w}(x,y,t)$ is obtained by differentiating Eq. (5) as

$$\ddot{w}(x,y,t)=\sum_r\frac{f_r\left(x_0,y_0\right)f_r(x,y)}{m_r w_r^*}\left[A_1(t)+A_2(t)+A_3(t)\right]\qquad (7)$$

where

(i) for $0\le t\le T_H$

$$A_1(t)=-\frac{F_0 w_r}{2}e^{-x_r w_r t}\sin\left(w_r^* t-\Theta_1\right)$$

$$A_2(t)=\frac{F_0 w_0^2}{\Omega_2}\sin\left(2w_0 t+\Theta_2\right)+\frac{F_0 w_r^2}{4\Omega_2}e^{-x_r w_r t}\sin\left(w_r^* t-2\Theta_1+\Theta_2\right)$$

$$A_3(t)=-\frac{F_0 w_0^2}{\Omega_3}\sin\left(2w_0 t-\Theta_3\right)+\frac{F_0 w_r^2}{4\Omega_3}e^{-x_r w_r t}\sin\left(w_r^* t-2\Theta_1+\Theta_3\right)$$

(ii) for $t>T_H$

$$A_1(t)=\frac{F_0 w_r}{2}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)-\Theta_1\right]-\frac{F_0 w_r}{2}e^{-x_r w_r t}\sin\left(w_r^* t-\Theta_1\right)$$

$$A_2(t)=-\frac{F_0 w_r^2}{4\Omega_2}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)-2\Theta_1+\Theta_2\right]+\frac{F_0 w_r^2}{4\Omega_2}e^{-x_r w_r t}\sin\left(w_r^* t-2\Theta_1+\Theta_2\right)$$

$$A_3(t)=-\frac{F_0 w_r^2}{4\Omega_3}e^{-x_r w_r (t-T_H)}\sin\left[w_r^*\left(t-T_H\right)-2\Theta_1+\Theta_3\right]+\frac{F_0 w_r^2}{4\Omega_3}e^{-x_r w_r t}\sin\left(w_r^* t-2\Theta_1+\Theta_3\right)$$

NUMERICAL RESULTS

The numerical results are presented in the case of impact between a 24.7 mm diameter steel ball and an aluminum plate with dimensions of 480×360 mm and thickness of 2 mm which is clamped in a frame with thickness of 10 mm. A free layer damping material with thickness of 1 mm is applied to the reverse side of plate. The sphere initial velocity is 1.0 m/s and the impact occurs at the center of the plate. Numerical calculations have been carried out taking into account the 16 modes with natural frequencies up to 600 Hz. The undamped mode shapes and modal parameters were computed for the composite plate with the visco-elastic material treated as it were purely elastic, then the modal loss factors were obtained by the modal strain energy method [10]. The integration in the Eq. (6) is performed by dividing the plate into a lattice of 20 mm. Calculated displace and sound pressure waveforms at the center of the plate are shown in Fig. 3 for the plate without and with damping layer. The first peak observed at 1.5 millisecond in the waveform of sound pressure is due to rapid surface deformation of the plate and is followed by the decaying sound pressure due to pseudo-steady state radiation [1]. Damping reduces the sound radiation only from the pseudo-state vibration of the plate. Fourie spectra of the plate vibration velocity and sound pressure are given in Fig. 4. The displacement spectrum has been multiplied by jw to obtain the velocity spectrum. The peaks at natural frequencies apparently reduce by adding damping layer. The peak plate velocities and sound pressures in the frequency domain are plotted in Fig. 5 for various impact velocities. It can be seen that the sound pressures are proportional to the impact velocity.

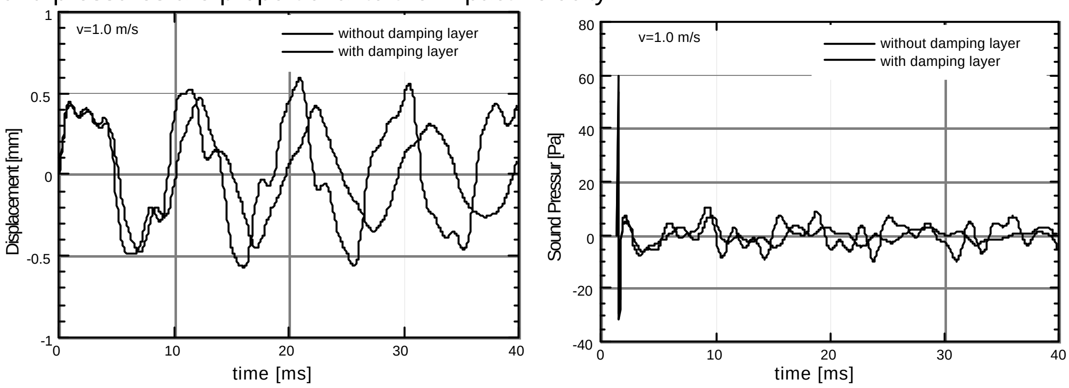

FIG. 3. Calculated plate displacement and sound pressure.

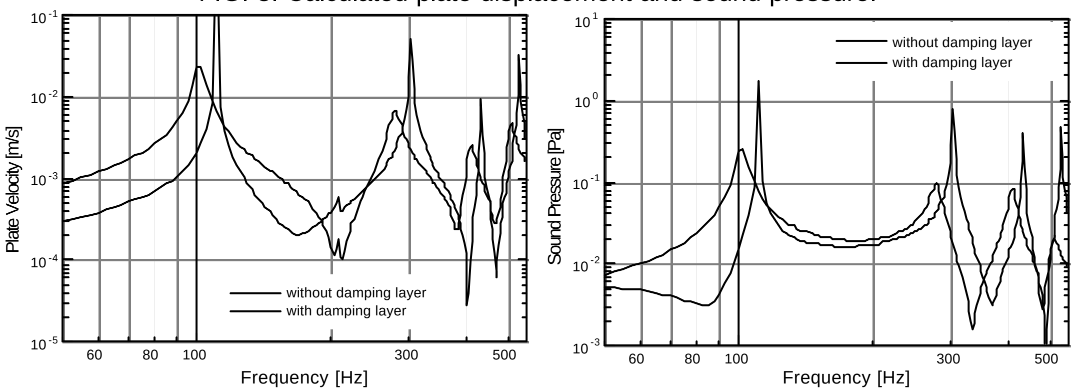

FIG. 4. Fourie spectra of calculated plate velocity and sound pressure.

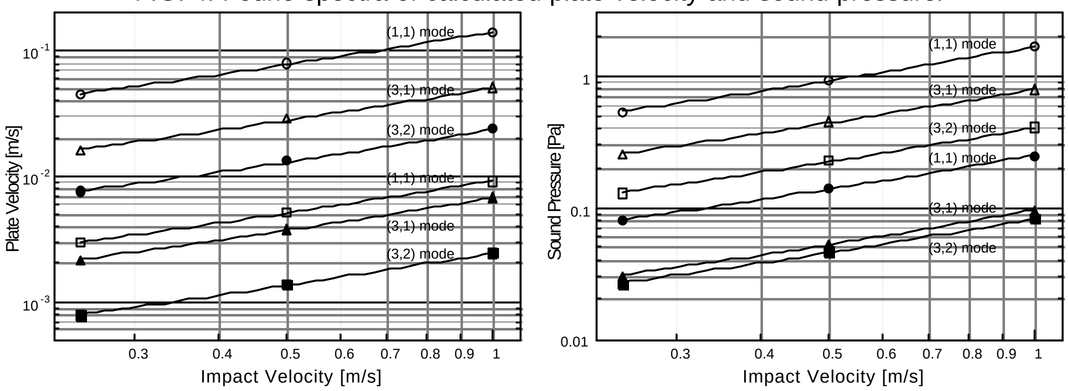

FIG. 5. Peak plate velocity and sound pressure vs. impact velocity.

EXPERIMENTS

Measurements corresponding to the theoretical prediction were carried out to obtain plate vibration responses and radiated sound pressure waveforms and their spectra for plates without and with damping layer. Rectangular aluminum plates were clamped at the edges using sandwich frames bolted together and were placed in a baffle. The schematic diagram of the experimental setup is shown in Fig. 6. The plates were impacted at the midpoints by a steel ball of 2.47 cm diameter. The ball was dropped from various heights to obtain different impact velocity. The plate displacement and sound pressure waveforms were measured using a laser displacement meter and sound level meter.

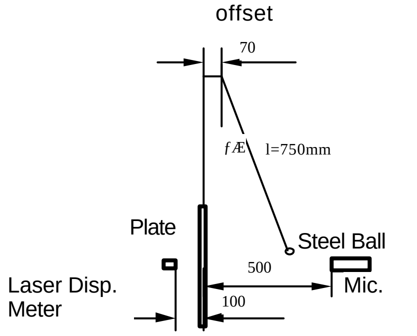

FIG. 6. Experimental setup.

Examples of measured plate displacement and sound pressure waveforms are given in Fig. 7. The spectra for the experimental results are shown in Fig. 8 [11]. The measured waveforms and spectra are similar to those analytically obtained. Peak plate velocities and sound pressures at frequencies corresponding to modes (1,1), (3,1), (3,2) are plotted against the impact velocity in Fig. 9. The sound pressures are approximately proportional to the impact velocity.

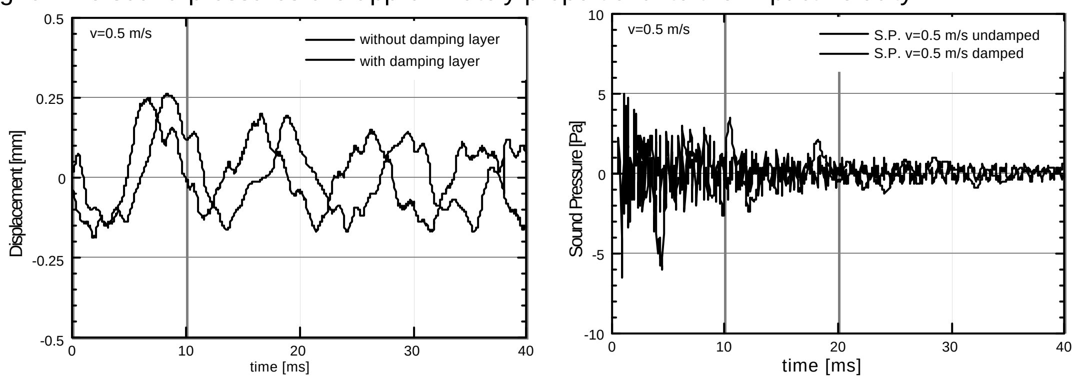

FIG. 7. Measured plate displacement and sound pressure.

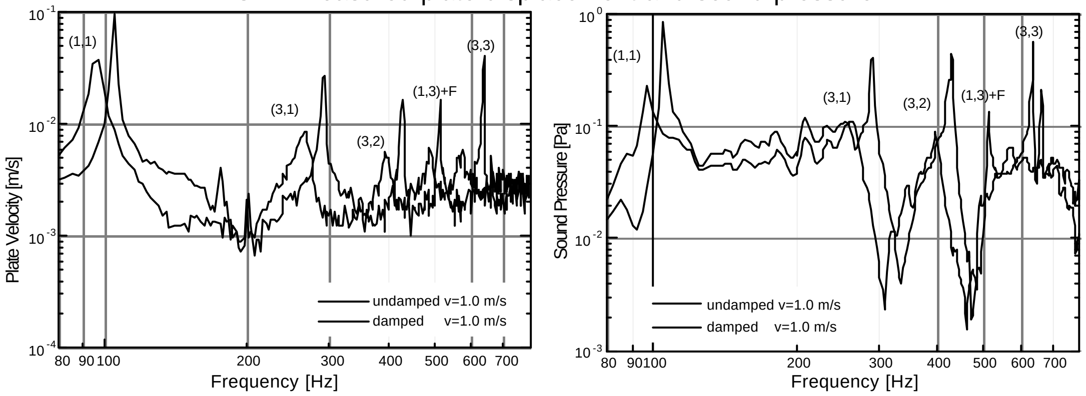

FIG. 8. Fourie spectra of measured plate velocity and sound pressure.

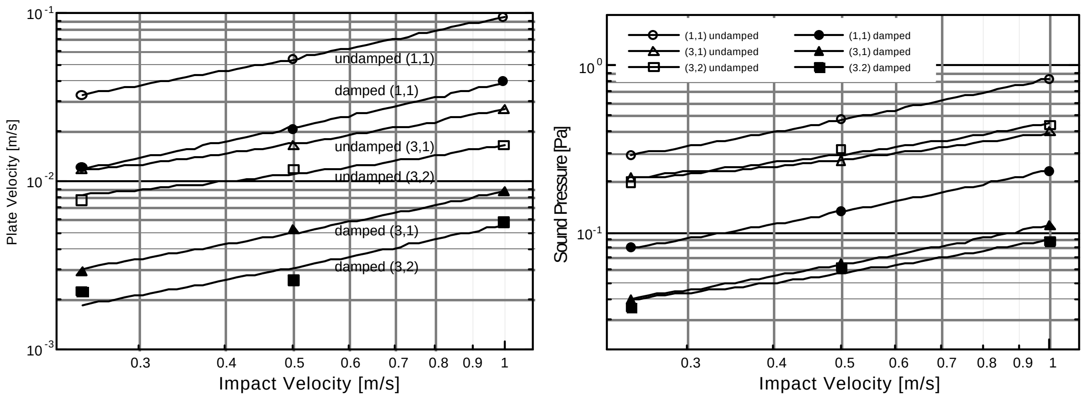

FIG. 9. Peak plate velocity and sound pressure vs. impact velocity.

CONCLUSIONS

Transient sound radiation from baffled rectangular plates with and without free viscoelastic layers excited by an impulsive point force has been obtained analytically and validated experimentally. Consistent results of the plate displacement and radiated sound pressure between the predicted and measured are obtained. The radiated sound pressure is proportional to the impact velocity. The use of viscoelastic material has the effect of reducing the spectral peaks of sound pressure at the natural frequencies of plate vibration.

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