DYNAMIC MODELLING OF SOME NONLINEAR MATERIALS

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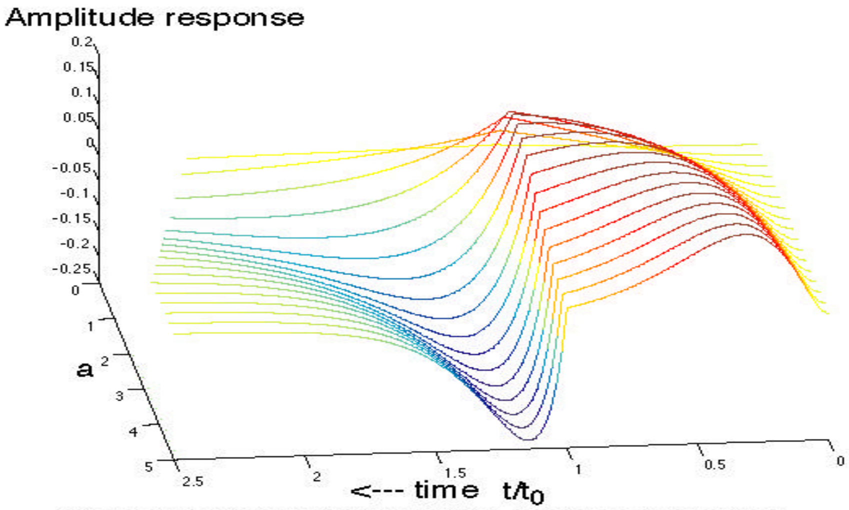
ABSTRACT

A nonlinear layer can be a model for a cloud of gas bubbles in a liquid, a crack or split plane in a solid, or contact between two tighted surfaces. Solutions were derived for media under strong load.
Numerical calculations based on Preisach-Mayergoyz space description (nonlinear stress-strain relationships typical for solids containing mesoscopic inhomogeneities or defects) have given results like those obtained from LISA (see results by Delsanto).
Non-uniform massdistribution of grains immersed into a vibrating fluid create internal forces which is responsible for generation of higher harmonics.
Tests on slow dynamics were performed.

PULSE PROPAGATION THROUGH NONLINEAR LAYERS

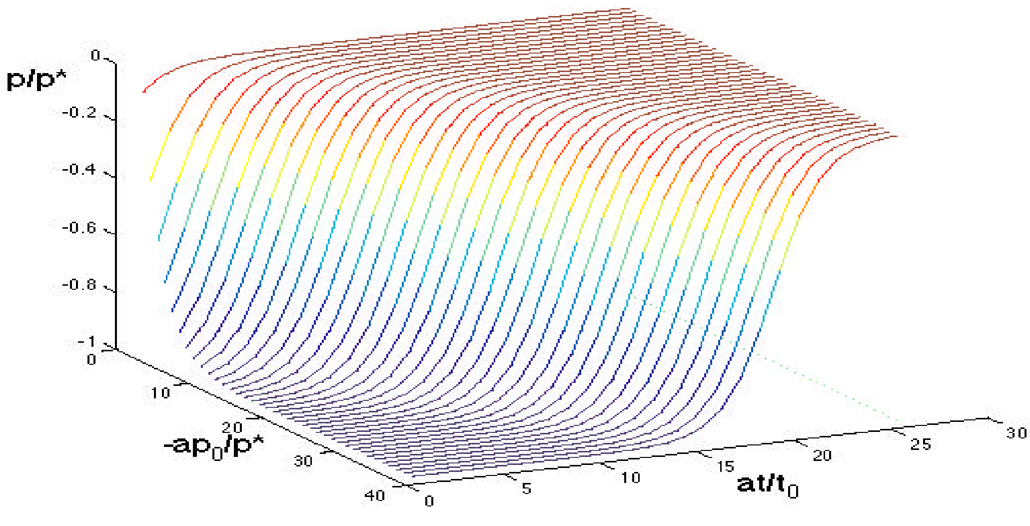
The problem of normal incidence of a plane wave on a layer is one of the simplest and most important problems in the acoustics of layered media. It attracts considerable interest for two reasons. First, it is rather simple, and one can look forward to obtaining the solution, which must be in analytical form. Second, the layer can serve as a model of a real nonlinear inclusion such as a bubble cloud in a liquid, or a region inside a solid with a high content of defects, or a resonant cavity in concrete, or a geological structure.
The statement of the problem is a plane layer with density ρ_0 and sound velocity c_0 is plaved between x=0 and x=h. This layer is surrounded by another medium with density ρ_1 and sound velocity c_1.

In the figure below is shown the additional nonlinear response from a monopolar rectangular incident pulse, the parameter $a=2c_0^2t_0\rho_0/(c_1\rho_1h)$. [1]



Additional nonlinear response from a nonlinear layer to a rectangular incident pulse. The parameter a is dependent on the ratio of pulse width over width of layer.

In next figure is shown the total (nonlinear) response from an incident negative δ-pulse, the parameter $a=2c_0^2t_0\rho_0/(c_1\rho_1h)$ (the same as above). In this case the nonlinear dependence between density and pressure is $\rho(p) = p^*/c_0^2 \ln(1+p/p^*)$. It is seen that the normalized pressure p/p^* can not be less than –1 which defines the value of p^*. This specific case has a analytical solution: $p/p^* = [(1-\exp(-ap_0/p^*))\exp(-at/t_0)]/[1-(1-\exp(-ap_0/p^*))\exp(-at/t_0)]$.



Strong nonlinear response from a nonlinear layer to a short rarefaction pulse in form of a negative δ-function. The maximum negative pressure that can be applied to the material is **p^***. The parameter **a** is depending on the length of the pulse divided by the width of the nonlinear layer, **$a \sim c_0t_0/h$**.

NUMERICAL MODELLING OF MESOSCOPIC INHOMOGENEOUS MATERIALS

The P-M space phenomenological model [2], describes an assemblage of elastically elements called hysteretic mesoscopic units (HMU) correspond to the elastic bond system in NME materials. The elastic behaviour of these units are compared to atomic elasticity, very complex and difficult to explain. But P-M space describes an assembly containing many units and therefore some simplification can be done about the behaviour of these units. The element will close and open at different lengths depending on if stress is decreasing or increasing. Comparing the stresses when the element is loaded and unloaded in a graph, one gets the P-M space graph.
The structure in a material can be considered in different levels. For instance there is an atomic structure and a crystal structure. To calculate the mechanical behaviour of a material is extremely time consuming when considering such small scales as the atomic structure level. The size of the calculation would be enormous and there would also be problems with the definition of several different forces that acts on the atomic structure.

But there is also a structure considering the grains in the material, which is called mesoscopic. These grains in the material are defined by the arrangement of the atoms. The atomic arrangement will be exactly the same in one specific grain. But the orientation of the atoms in adjoining grains is different.
Because of the difference of the behaviour between the grain and the grain boundary it is of interest to control the size and the density of the grains. Change these parameters and you get a different material property. Reducing the size of the grain there will be more grains and of course more boundary that obstructs the movement of a dislocation. Which leads to an increasing strength of the material.
To be able to calculate the wave propagation in a NME material it is necessary to make some assumptions considering the behaviour of the grains. They are: the Young's modulus is constant for all the grains; deformations are only considered in one dimension, which means that the phenomena of lateral contraction will not be considered; all the grains have the same width; and all the grains are subjected to the same stress at the same time. [3]. The most complex part is to describe the bond system. Many factors influence the behaviour of the bond system and it is impossible to pay attention to every one of these factors. It would demand an enormous amount of computer capacity to be able to consider every influence on the grains [4]. Because of the small forces and the large amount of data that have to be considered, the internal forces and microstructure are assumed to not influence the behaviour of the bond system. The external conditions are also considered to not change the behaviour. The model is based on the macrostructure and the bond system will be influenced from forces that act in a macrostructure. To be able to explain the behaviour of a material one have to explain, or in some way determine, the relationship between external forces and the internal behaviour of the material. The most common and satisfying way of explaining the behaviour of a material is to consider the relation between stress and strain. As has been mentioned previously the grains will be modelled as perfectly elastic springs, which means that the nonlinear mesoscopic elastic behaviour of the model will be introduced in the model through the bond system.

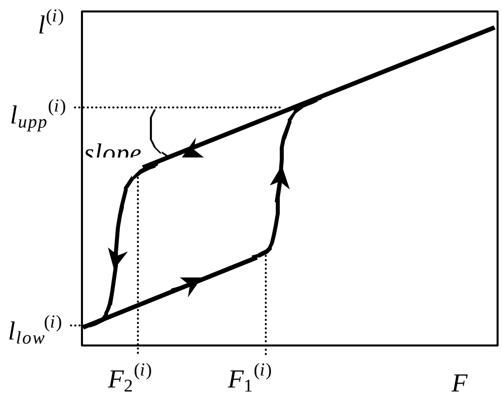


Figure 1. How the length of interstice i depends on the traction force F.

The rod is first discretized as into a large number of tiny segments or masses whit length $\Delta l = (L - l_{int})/n$ where l_{int} is the length of the interstices when the rod is unloaded and n is the number of segments the rod is discretized into. The segments are then divided into grains consisting of one or more segments. A PM-space model can be seen in figure 2. [3]

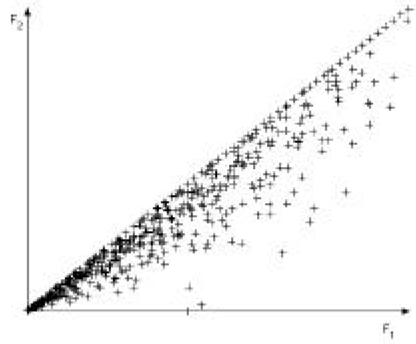


Figure 2 . PM-space .

In the stress/strain graph the stress is calculated as $s = F/A_{rod}$, where A_{rod} is the area of the rod. In Figure 3 left has been used different slopes of the interstices.

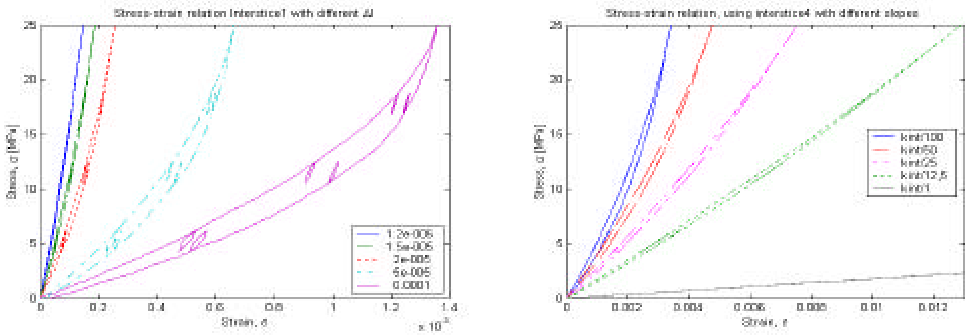


Figure 3. Stress-strain relation using different slopes of the interstices (left) and for different opening and closing lengths of the interstices (right).

In figure 3 right different elongations (l_{max} - l_{min}) of the interstices has been used. It shows that as the elongation of the interstice gets higher the nonlinearity gets higher. These results are very similar to the results obtianed by Delsanto with the LISA software. [5]

NONLINEAR DYNAMICS OF GRAINS IN A LIQUID-SATURATED SOIL

Known models with huge nonlinear response of grainy media are based on nonlinear stress-strain relationships typical for solids containing mesoscopic inhomogeneities or defects in their structure. Such nonlinear behaviour is well-defined at significant local deformation caused by high applied load, even if the load does not vary in time. However, there exist a different type of nonlinearity which manifests itself due to inertial forces between grains. Such forces appear in a moving noninertial frame of reference, in particular, if a small system of interacting particles is placed into a vibrating fluid. High spatial gradients of internal forces are determined by the non-uniform distribution of mass. These gradients varying in time can excite the internal degrees of freedom. So, even at harmonic vibration of the fluid caused by sound, the nonlinear internal dynamics can be responsible for generation of higher harmonics.
A model of grainy medium is developed as an example of nonlinearity of inertial type. The model deals with an ensemble of grains immersed into vibrating fluid. The inertial attractive forces have hydrodynamic origin, and the repulsive forces are caused by deformation of colliding grains.

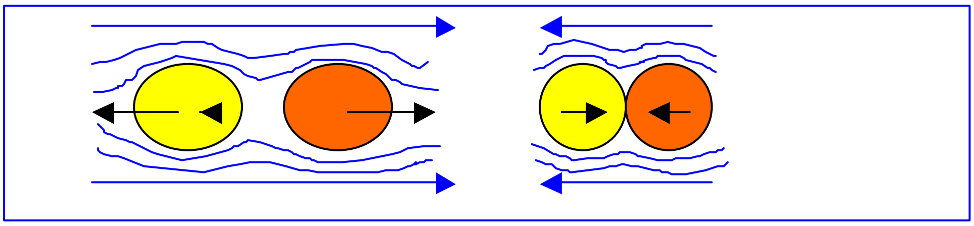
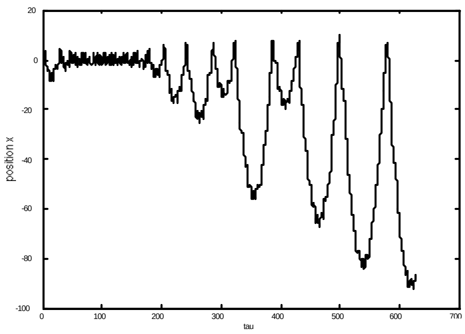


Figure 4. Two colliding grains in a fluid-saturated medium.

Several interesting phenomena were observed at the computer modelling of this nonlinear dynamic model, including a huge higher harmonics generation. For some magnitudes of parameters, the effect of appearence of low-frequency vibration is well-pronounced (Figure 5). It is interesting that these low frequencies do not depend on the frequency of acoustic wave. This regime is accompanied by stochastic oscillations caused by random collisions of particles having unpredictable velocities. These results are shown in the following figures containing temporal behaviour of model (the process of the development of forced highly nonlinear vibrations), as well as the corresponding spectral distribution.



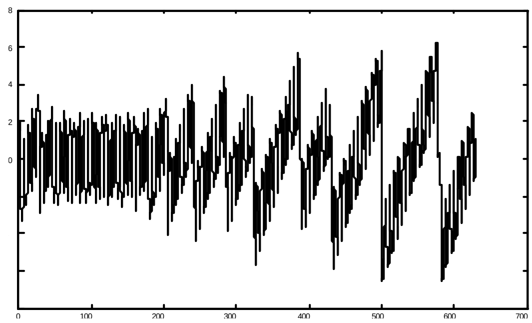
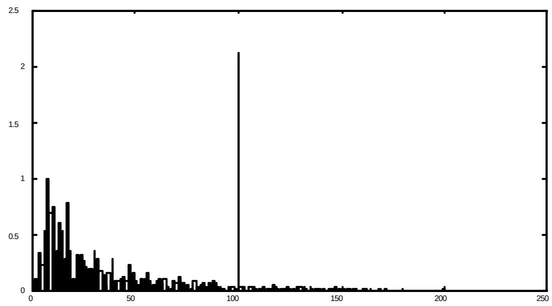


Figure 5. Distance between two grains (top); relative velocity (upper right); and the frequency content of the velocity (right).



EXPERIMENTS ON NONLINEAR NONDESTRUCTIVE TESTING

By using a Nonlinear Wave Slope Amplifier method the time signal response from samples can be used to indicate the nonlinearity in the materials connected with the presence of cracks. Below in Figure 6 is shown how the difference between an undamaged material on top differs markedly from the response from the damaged one below.

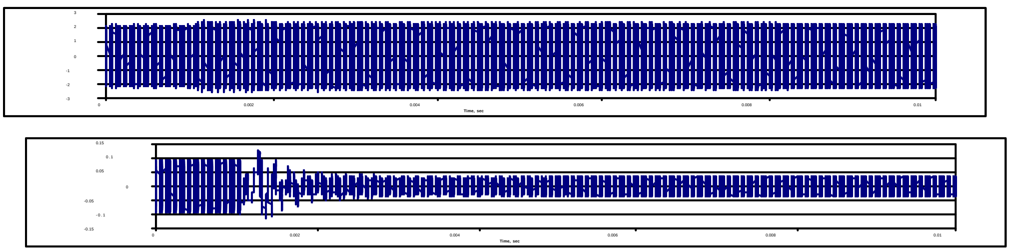
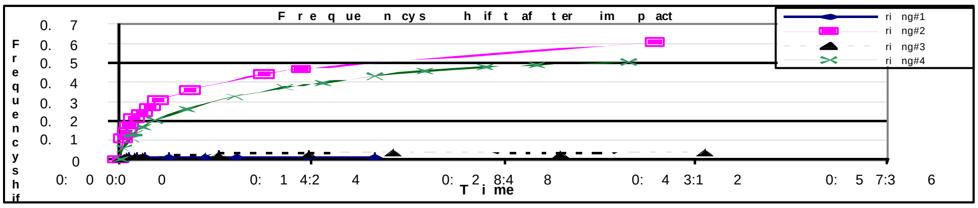


Figure 6. The huge difference between the nonlinear response of an undamaged (top) and a damaged (bottom) metallic ring.

As can be seen the effect seem is immediate and but recovers more slowly. In fact this is an entirely new phenomena, that the material state (damping and elastic modulus) changes for up to hours after some conditioning. This can be measured by for example resonance frequency monitoring as i seen in Figure 7 . It can not be described by the PM space model above. For some time it was believed that it was due to phenomena on the mesoscopic level,[6] but indeed, late discoveries indicate that the phenomena are *not* on the mesoscopic scale, but smaller – probably atomic, in origin. This phenomena is called *Slow Dynamics* (SD). A 'large' amplitude wave alter temporarily the material through which it passes ! Of great consequence is that SD can be used for nondestructive, noninvasive diagnosing and monitoring of damage, disbonding etc. *In fact, there is no more sensitive diagnostic of material "damage" than SD to our knowledge*.[7]



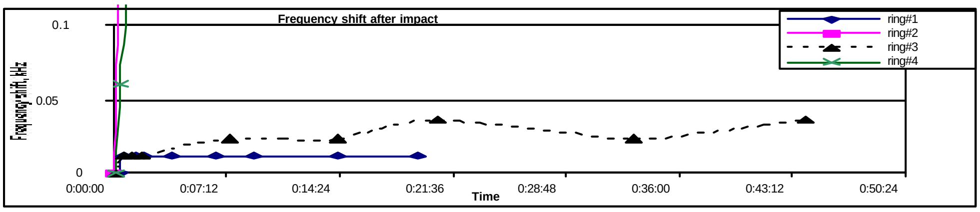


Figure 7. Slow dynamics measured by resonance monitoring for two damaged and two undamaged samples.

Acknowledgements

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References

1. Hedberg, C.M. and Rudenko, O.V., Pulse response of a nonlinear layer, J. Acoust. Soc. Am. 110(5), 2340-2350 (2001).
2. Guyer, R.A., McCall, K.R., Boitnott, G.N., (1995), Phys. Rev. Lett. 74.
3. Carlsson, J. , Johansson, M.and Olofsson, J. Modelling of damaged material and experimental pulse propagation through a nonlinear layer, Master Thesis Mechanical Engineering, Blekinge Institute of Technology (2002)
4. Askeland, D.R., (1998), The Science and Engineering of materials, Stanley Thornes, Cheltenham UK.
5. Delsanto P.P. et al., Wave Motion 26, 329-339 (1997); J.Acoust.Soc.Am 104, 1-8 (1998).
6. Guyer R.A., Johnson P.A.. Nonlinear mesoscopic elasticity: evidence for a new class of materials. Physics Today, April 1999, p. 30-36.
7. Johnson, P.A., Sutin. A., Darling, T., Nonlinear, elastic wave slow dynamics in certain metals, in a sintered ceramic, and in cracked solids: a probe of atomic-scale mechanics, Nature, in review, 12/2001.