MODELLING OF THE TYRE RADIATION OVER ABSORBING SURFACES USING THE EQUIVALENT SOURCES METHOD

PACS REFERENCE: 43.40.Rj. Radiation from vibrating structures into fluid media.

Bécot François-Xavier(1); Kropp Wolfgang(2)
(1) Transport and Environment Laboratory, INRETS
(2) Department of Applied Acoustics, Chalmers University of technology
Case 24, 25 Av. Mitterrand
69675 BRON Cedex
Tel: int+33 472 14 24 10
Fax: int+33 472 37 68 37
becot@inrets.fr

ABSTRACT

A model to predict the tyre radiation over absorbing road surfaces is presented. The two aspects of the modelling, the tyre radiation and the ground effects reproduction, are tackled using the equivalent sources method. Combining the features of both models allows to study the influence on the tyre noise emission of road surfaces having complex acoustical properties. It results in an original and a low-computationally demanding implementation compared to approaches using integral equations. Numerical results are presented in the case of totally soft surfaces and finite impedance surfaces. This preliminary study prooves the applicability of the equivalent sources method for the prediction of traffic noise in presence of absorbing surfaces.

INTRODUCTION

The correct prediction of the traffic noise has become an important issue of nowadays research. This effort has been leading to many different approaches for the tyre noise prediction. Some of them ended up in practical solutions and are now being tested in-situ. However, their extension to large scale experiments, which are needed for a true traffic noise reduction, is limited by the lack of optimisation procedures. In this context, the present work is expected to provide an accurate and low-computational numerical tool for the prediction of tyre noise.
This study is based on the approach proposed in [1,2] for the prediction of the tyre noise radiation. This model uses the principle of the equivalent sources (ES) method, also referred to in literature as source simulation technique, to reproduce the sound field due to a vibrating tyre. Moreover, it was shown in [3] that the ES method applies successfully to reproduce the ground effects during the noise propagation over finite impedance surfaces. In this paper, the combination of boths approaches is proposed to model the influence of absorbing surfaces on the tyre / road contact noise.
After the principle of the ES method being reminded, the model for the tyre radiation is presented. A technique to introduce the ground effects is then exposed, as well as its main feature. Finally preliminary results are presented to proove the applicability of the ES method for the prediction of the tyre noise in presence of absorbing road surfaces.

TYRE RADIATION MODELLING

Multipole model

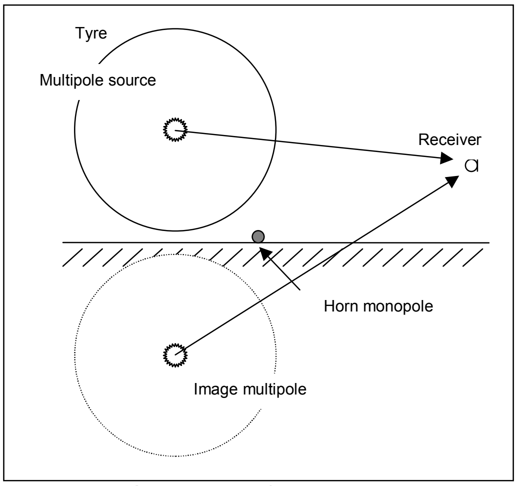


Figure 1 : Geometry of the multipole model

The ES method is an alternative to integral equations based methods for the prediction of the sound field radiated by vibrating objects of arbitrary shape. The main idea is to reproduce the emitted field by a finite set of sources placed inside the body of the radiator.
In our case, the tyre, modelled as an infinite cylinder, is replaced by a multipole acoustical source, giving the name of multipole model. In other words, assuming a time harmonic signal, the pressure field is expressed by the superposition of an infinite number of modes.
The effects of an infinite road surface are included using an image source placed symmetrically aside the ground plane (cf. Figure 1). Its amplitude is R times the amplitude of the multipole above the road surface.

Omitting the time dependency $e^{j\omega t}$ here and in the following, the pressure field in the half space above the road can be written

$$p(x_r) = p_m(r_1, \varphi_1) + p_m(r_2, \varphi_2) = \sum_{m=-N_{\max}}^{m=+N_{\max}} A_m \left[H_m^{(2)}(kr_1) e^{jm\varphi_1} + R.H_m^{(2)}(kr_2) e^{jm\varphi_2} \right] \qquad (1)$$

where (r_1, φ_1) and (r_2, φ_2) are the position of the receiver x_r seen from the multipole and the image multipole. $H_m^{(2)}$ is the Hankel function, second kind, of order m. A_m is the amplitude of the elementary vibration mode of order m. Practically, only a finite number of modes is included and N_{max} in the equation (1) denotes the highest vibrational mode taken in.

Boundary Condition On The Tyre Surface

In the application of the ES method, the unknown amplitudes A_m are determined from the boundary condition. In the case of the tyre noise radiation, a prescribed particle velocity on the tyre belt is given by a monopole source, the "horn monopole", placed inside the horn-like region. The better the boundary condition is fulfilled, the better the prediction will be. The key point of the method is that both multipoles, the "real" and the image multipole, must together fulfill the boundary condition.
Once the modal amplitudes determined, the pressure field at any point above the road surface and outside the tyre body can be calculated. The obtained pressure field accounts for the presence of a rigid plane surface, for instance, if $R = 1$. Even though it is not mathematically correct, this model can give tendancies for absorbing surfaces if R takes on the values of a plane wave reflection factor [2].

Influence of the reflection coefficient on the horn effect amplification factors

The tyre alone is not a good radiator [1] but the close proximity of the road increases its radiation efficiency. This phenomenon, known as horn effect, results in a sound amplification that can reach 20 dB in certain cases. This phenomenon is very important because it also leads to a change in the tyre radiation directivity. The correct prediction of these amplification factors is a good criteria for the validation of the multipole model.
As an example, the amplification factors for different heights of the tyre above a totally rigid surface are shown in Figure 2. It is found that, when lifting up the tyre, the maximum of amplification, besides being reduced, is shifted towards the low frequency range. This could explain the often measured result that rolling on very rough road leads to a lower radiated noise

at high frequencies. This may be thought of as a mean of combining water draining capacities and low radiated noise.

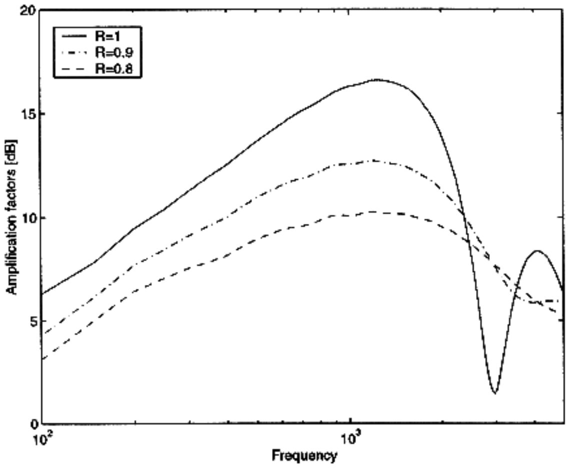


Figure 2: Influence of the reflection coefficient R

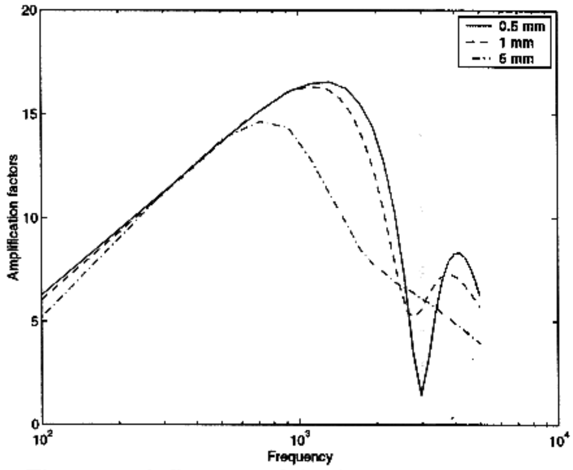


Figure 3 : Influence of the height of the tyre

The influence of the value of a real reflection coefficient is examined in Figure 3. When the absorption of energy by the road increases (R<1), the maximum level of amplification decreases. Furthermore, the inteference pattern is no longer observed and the horn effect vanishes.

Results in [2] showed that the horn effect amplifications measured in presence of absorbing surfaces might be approached by using a reflection factor for plane waves. This solution is however not correct as a reflection coefficient for cylindrical waves is expected. Therefore, the equivalent sources method has been applied to the prediction of the noise propagation over inhomogeneous absorbing surfaces. The method, presented in [3], is summarized in the next paragraph.

INTRODUCTION OF THE GROUND EFFECTS

Boundary Value Problem

The geometry is also two-dimensional. A source of arbitrary order is radiating over an inhomogeneous surface of arbitrary acoustical impedance. A continuous layer of sources is placed on the ground to account for the reflections on the ground surface. The total sound field is thus the superposition of the incident field and the fields from all the ground sources:

$$p(x_r) = p_{inc}(x_r) + \int_\Gamma Q(\xi) G(\xi | x_r) d\xi \qquad (2)$$

$Q(\xi)$ is the amplitude of the source located at a point ξ of the surface Γ. $G(\xi|x_r)$ is the free space Green's function for a ground source located in ξ calculated at a point x_r. In our case, a layer of monopole sources is chosen, so that $G(\xi|x_r) = -\frac{j}{4} H_0^{(2)}(k|x_r - \xi|)$. The velocity field in the direction $\vec{n}$ due to the sources in presence can be expressed using the corresponding velocity Green's functions $G^{(v,n)}(\xi|x_r)$:

$$v(x_r) = v_{inc}(x_r) + \int_\Gamma Q(\xi) G^{(v,n)}(\xi | x_r) d\xi \qquad (3)$$

At a point x of the ground surface, pressure and velocity are connected by the value of the normal acoustical impedance at this point $Z_n(x)$. After rearranging the terms, a boundary value problem is formed :

$$\int_{\Gamma} Q(\xi)\left[G(\xi|x) - Z(x)G^{(v,n)}(\xi|x)\right]d\xi = Z(x).v_{inc}(x) - p_{inc}(x_r) \qquad (4)$$

This is an integral equation of unknown $Q(\xi)$. Once the amplitudes determined, the pressure from the system of sources can be calculated at any field points above the road surface using equation (2). One can also take advantage of particular Green functions that already account for the effects, e.g., of a totally rigid plane.

Handling of the singularities

The difficulty in inverting the integral equation (3) comes from the singularities included in the given Green's functions, each time the receiver x is at a source position ξ. A standard procedure consists in estimating the Cauchy principal value of the integral at these points. The contribution from the singularities is found to be of finite extent, allowing thus a numerical evaluation of the integral as long as the singular point itself is not chosen. For the calculation presented here, a Gauss-Legendre quadrature has been used.
It was proved in [3] that this method can be applied to predict the sound field over inhomogeneous ground. The main feature of this technique is to allow the incident source to be of any kind (strongly directional or not), which makes it attractive for the domain of the traffic noise.

Preliminary Results

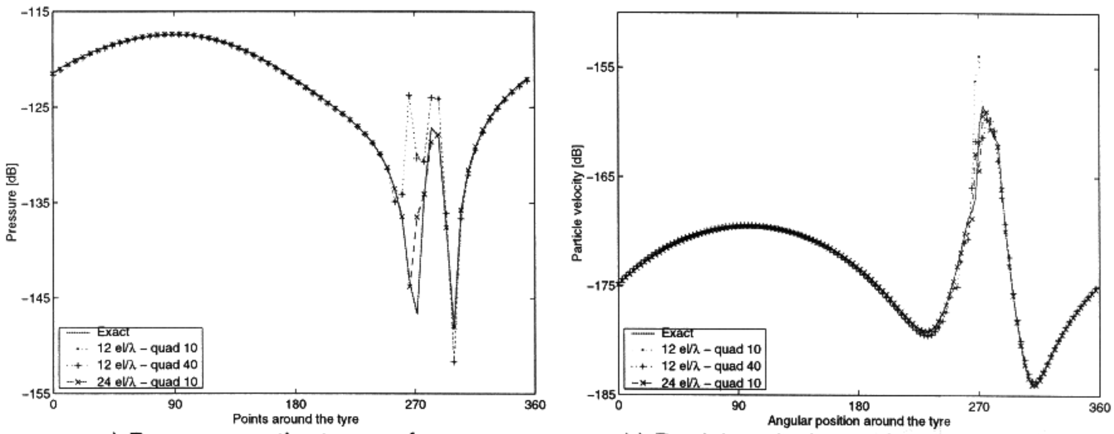
In order to proove the applicability of the previous technique to the case of a radiating tyre, a preliminar calculation is presented here. The objective is to calculate the sound field from a radiating tyre in presence of a totally soft ground plane.
First, the sound field from a tyre is computed in presence of a totally rigid road surface. As previously, the boundary condition on the tyre is given by the field due to the horn monopole. In a second step, this field is assumed to impinge on the totally soft surface. The ground sources are then "switched on" to fulfill the required impedance (Z_n = 0). The reflected sound field is finally added to the incident one to obtain the total pressure field :

$$p_{tot}(x_r) = p_{inc}(x_r) + p_{ground}(x_r) = p_{m_1}(x_r) + p_{m_2}(x_r) + p_{mono}(x_r) + p_{ground}(x_r) \qquad (5)$$

By doing so, the ground sources are expected to have their strongest influence on the total sound field, which is a good challenge to test the method. Moreover, the resulting pressure field can be compared to the exact solution available in this case :

$$p_{exact}(x_r) = p_{m_1}(x_r) - p_{m_2}(x_r) + p_{dipo}(x_r) \qquad (6)$$

where p_{dipo} is the pressure from the horn monopole embedded in a soft baffle. One can show that the resulting sound field corresponds to the pressure of a dipole at the same position.



a) Pressure on the tyre surface
N_{max}=32 ; f = 200 Hz

b) Particle velocity on the tyre surface
N_{max}=64 ; f = 1500 Hz

Figure 4 : Field from a vibrating tyre 1mm above a totally soft surface – tyre height = 1 mm.

In Figure 4 a) and b), are shown results of this procedure for the pressure and the velocity fields. Two type of multipoles are tested at two different frequencies. In the legend, the first cited parameter is the number of elements per wavelength (12 or 24 in the calculations shown here). The second parameter is the order of the quadrature used for the evaluation of the ground sources contribution (10-th or 40-th order quadrature). On these figures, 0° corresponds to the point the most right on the tyre surface and 270° to the closest point to the ground surface.
One can observe that in both situations, the exact fields can be fairly well predicted provided an adapted ground discretization. Not surprisingly, the main difficulties to achieve the correct field occur for the points in the contact zone. In this region, the fields variation are important and require a very fine discretization of the ground surface. Moreover, the order of the quadrature seems to have a weaker effect on the accuracy than the number of elements per wavelength. This may be due to the fact that the higher the order of the quadrature, the closer to the singularity. In this case, the numerical evaluation of the Hankel functions may not be accurate because of very small values of the argument. The gain in the ground description may thus be deteriorated by these numerical instabilities.

According to these preliminary results, the ES method seems adapted to predict the noise radiation due to a vibrating tyre over energy absorbing surfaces. In the next paragraph, the implementation of such an improved multipole model is presented.

IMPROVED MULTIPOLE MODEL

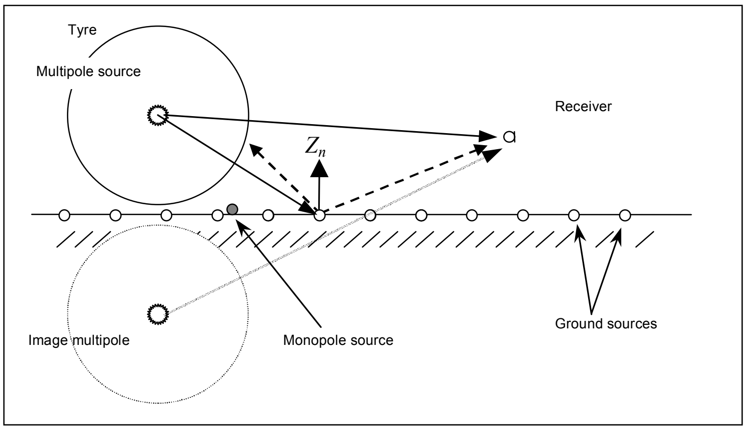


Figure 6 : Geometry of the improved multipole model.

The system of sources of the improved multipole model results from the combination of both previous models. It is shown on Figure 6. It comprises the multipole source and its image for the tyre radiation. The ground effects are accounted for by placing a number of simple sources at the ground level.

Finally the horn monopole gives the prescribed velocity boundary condition on the tyre surface. On the ground surface, a normal impedance distribution is given, which gives the second boundary condition of the problem.

<u>Implementation</u>

The key point of the implementation is that the multipole sources together with the ground sources must fulfill the two boundary conditions. The unknown amplitudes of the problem are the multipole source strengths A_m (cf. equation (1)) and the ground sources amplitudes $Q(\xi)$ (cf. equation (2)). For their determination, the required transfer functions are homogeneous to a pressure and a velocity, from the multipole to the ground sources on one hand, and from the ground sources to the tyre surface on the other hand. Due to a too different scaling of these matrices, the direct inversion of the system cannot be done accurately.
To overcome this, an iterative process is implemented, which reproduces the multiple reflexions occuring in the horn region. First, the multipole amplitudes are computed to include a totally rigid plane. In a second step, the ground sources are switched on to compensate for the different impedance value at the ground surface. These latter sources create a perturbation field at the

tyre surface. In a third step, additional multipole amplitudes are computed to compensate this effect and thus to achieve the desired velocity distribution on the tyre surface. In turn, the new multipole sources create additional perturbation on the ground surface so that the impedance distribution may not be achieved. Correction for the ground sources are then computed and added to the previously determined amplitudes. The iterative process goes on until one of the perturbation becomes negligible.

Preliminary results

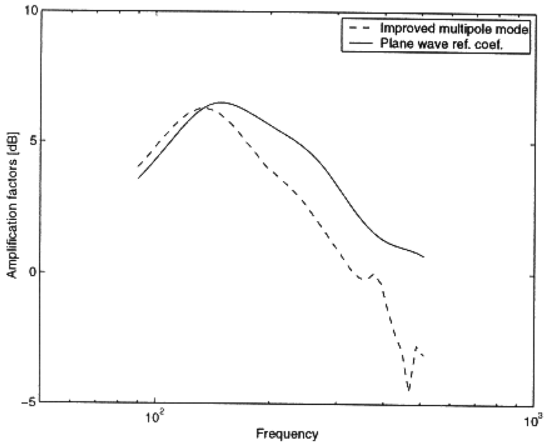


Figure 6 : Horn effect amplifications over absorbing surfaces - tyre height = 20 cm.

The technique presented above is implemented to compute the amplification factors over an absorbing surface (mineral wool here). For low height of the tyre, numerical instabilities make the convergence difficult to achieve. The work is under progress to solve this problem.
Therefore, are shown here amplification factors over an absorbing surface for a tyre at 20 cm above the surface. In this case, the convergence is achieved after 2 or 3 iterations. Results from the improved multipole model are compared to the original multipole model using a plane wave reflection factor, which tends to overestimate the horn effect predictions [2].
The good tendancies of the results proove the reliability of the method. However, this situation cannot be found in reality and the validation of the model is still to be completed.

CONCLUDING REMARKS

The ES method is a powerful tool to model the radiation and the scattering of sound. It was shown in previous studies that this technique successfully applies to the case of a radiating tyre and to the noise propagation over finite impedance surfaces. Preliminary results presented in this paper proove the applicability of the ES method to the case of a radiating tyre over asborbing surfaces. The complete validation of this approach is the subject of ongoing work.

ACKNOWLEDGEMENTS

The author would like to thank J.-F. Hamet from INRETS for the fruitfull discussions about the implementation of the improved multipole model.

REFERENCES

[1] W. Kropp, Ein Model zur Beschreibung des Rollgeräusches eines unprofilierten Guertlreifens auf rauher Strassenoberfläche, PhD thesis, Institut für Technische Akustik, Berlin, Germany, 1992.

[2] W. Kropp, F.-X. Bécot, S. Barrelet, On the sound radiation from tyres, Acustica-Acta Acustica, **86**:769-779, 2000.

[3] F.-X. Bécot, P. J. Thorsson, W. Kropp, Noise propagation over inhomogeneous ground using an extended Rayleigh integral method, inter.noise 2001, The Hague, Netherlands, 2001.