SIMULATION OF THE SOUND RADIATION FROM WHEEL-LIKE STRUCTURES USING THE BOUNDARY ELEMENT METHOD

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ABSTRACT

In this paper the calculation of sound radiation from a simplified wheel model using a 3D-FEM/BEM calculation is investigated. The modes of vibration of the wheel model are calculated in ANSYS and the corresponding velocity distributions on the surface were transferred to a 3D-BEM program to calculate the sound radiation of these mode shapes. The 3D-BEM program was developed within the research project and a short survey of the applied BEM routines will be given, as well as a description of the test procedure for the verification of the numerical results.

INTRODUCTION

The simulation of the sound radiation from wheel-like structures is the first result of a research project, that concentrates on developing a simulation tool for the optimisation of noise control applications, e.g. acoustic barriers. The project aims at the development of a 3D-BEM MATLAB® toolbox, that makes the calculation of the sound radiation of arbitrary structures available, as well as the calculation of the sound propagation above a ground of finite impedance and an acoustic barrier. The development of this toolbox follows a modular conception, which allows the permanent expansion of the program or the exchange of procedures. The formulation of an 3D-BEM code for the approximation of the sound radiation is the first step in this process.

THEORY

Starting point of the BEM calculation is the Helmholtz integral equation for exterior field problems,

$$\iint_s [\, p(y) \frac{g(x,y)}{\partial n(y)} - \frac{\partial p(y)}{\partial n(y)} g(x,y)] ds = C(x)\, p(x)\,, \tag{1}$$

where

$$g(x,y) = \frac{e^{-jkr}}{4\pi r} \quad \text{and} \quad r = r(x,y) = \| y - x \| \tag{2}$$

is the Green's function in the three dimensional free space, and $C(x)$ is 1 for x in the exterior domain, ½ for x on the surface of the radiating structure and 0 for x in the interior domain. A 3D-direct BEM code was developed to solve the Helmholtz equation on the surface of the structure for a Neumann boundary condition [$\partial p(y) / \partial n(y)$ is prescribed on the surface of the structure] and in the exterior domain. The fundamental derivations of the boundary element method in acoustics can be found for example in [2, 7, 8].
The surface of the structure is represented by a set of constant elements, i.e. the conversion of the Helmholtz integral equation into a corresponding set of equations was derived by approximating the boundary function on each surface element by a constant value. This is not the favourite discretisation method in terms of accuracy, but it can be fast and easily implemented. The diagonal elements of the resulting matrices, which become singular, were treated as it is described in [7]. The Helmholtz integral equation for an exterior problem does not have an unique solution at the characteristic eigenfrequencies of the associated interior Dirichlet problem. The CHIEF method was chosen to remedy the non-uniqueness at these eigenfrequencies. A brief description of the CHIEF method can be found in [8]. Several test showed, that the CHIEF method provides satisfactory results only at the first characteristic eigenfrequencies.

THE TEST PROCEDURE

To prove the quality of the calculation the comparison of the calculated results with an analytical reference solution is necessary. A possibility to create an analytical reference solution for any desired structure is the multipole test method as it is described by Ochmann [6]. A radiating multipole source is placed inside the structure and the normal velocity on the structures surface, which is intended to be sound- transparent, is then calculated. This normal velocity is used as input data for the BEM-calculation. The calculated sound pressure on the surface of the structure as well as in the exterior domain can be directly compared to the analytical solution of the multipole source. The test was carried out using a monopole source as radiating sound source inside the structure. The normal velocity on the surface of the structure, defined by the multipole source, is

$$v_n = -\frac{1}{j\omega\rho}\frac{\partial p}{\partial r}, \tag{1}$$

where ψ is fundamental solution of the monopole source in three-dimensional free space

$$\psi = \frac{e^{-jkr}}{4\pi r}. \tag{2}$$

Introducing a weighting factor for the fundamental solution, that provides a normal velocity $v_n = 1$ in a distance r_0, leads to the expression for the sound pressure in r as analytical solution

$$p_{analyt}(r) = \frac{r_0^{\ 2}}{r}\frac{j\omega\rho}{(1 + jkr_0)} e^{-jk(r - r_0)}, \tag{3}$$

The error of the calculation can be defined generally as follows

$$E = \sqrt{\frac{\iint\limits_S | p_{analyt}(r) - p_{BEM}(r) |^2 ds}{\iint\limits_S | p_{analyt}(r) |^2 ds}}. \tag{4}$$

Assuming that the pressure is constant over a single surface element, the one point integration over the surface of the structure leads to

$$E_{surf} = \sqrt{\frac{\sum_{i=1}^{N\_elements} | p_{analyt}(r_i) - p_{BEM}(r_i) |^2 \; S_i}{\sum_{i=1}^{N\_elements} | p_{analyt}(r_i) |^2 \; S_i}} \qquad (5)$$

where r_i are the centroids and S_i the areas of the single elements.

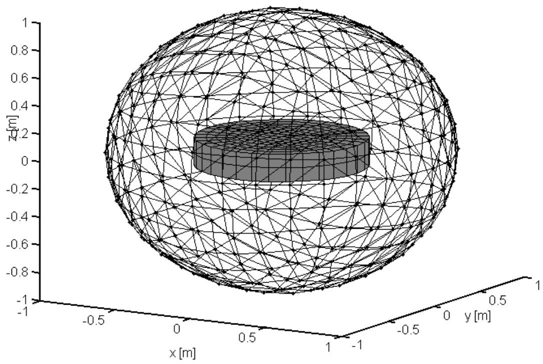

Fig.1: The radiating structure in a mesh of field points. The mesh, spread out by the field points, represents a sphere around the structure.

Regarding field points at r_i in the exterior domain, the radiation error integral E_{rad} was handled as an arithmetical mean, i.e. as a summation over the absolute error of field points, which form the mesh shown in Fig. 1

$$E_{rad} = \sqrt{\frac{\sum_{i=1}^{N\_fieldp.} | p_{analyt}(r_i) - p_{BEM}(r_i) |^2}{\sum_{i=1}^{N\_fieldp.} | p_{analyt}(r_i) |^2}} \qquad (6)$$

Fig. 2 shows the surface error and the radiation error for a wheel-like structure with a radius of 0.5m and a thickness of 0.1m. The frequency resolution is 10 Hz. Both errors are directly proportional to each other for the investigated structure. While the radiation error remains under a value of 0.05 over the considered frequency range, the surface error shows a slightly higher value.

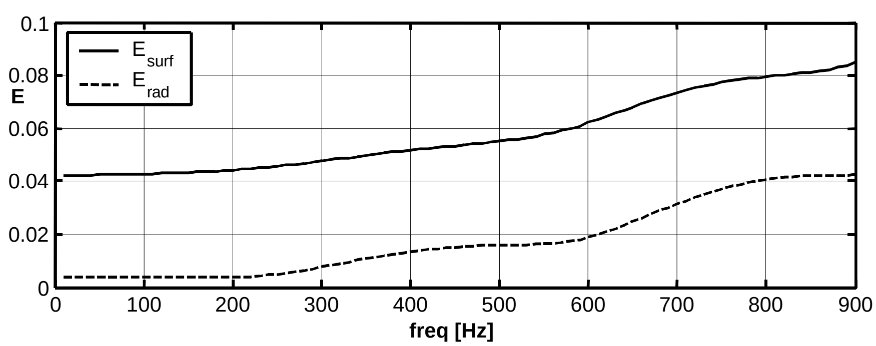

Fig. 2: Surface error E_{surf} (solid line) and radiation error E_{rad} (dashed line) in comparison.

It has to be noted, that the rule of thumb, whereby six linear boundary elements are required per acoustic wavelength, is fulfilled in the considered frequency range for the radiating structure as well as for the mesh, which is spread out by the field points. The surface error and the radiation error are obviously not directly proportional to each other. Fig. 3 shows the distribution of the relative error for each boundary element of the wheel at a frequency of 500 Hz. The qualitative distribution does not change with the frequency though the quantitative values increase. As it can be seen, the pressure at the edge between the cylindrical and the circular area of the wheel can not approximated sufficiently by the numerical calculation. A structure, which is more similar to a sphere, should show a minor surface error. This assumption was proven by calculating the monopole error for a very thick wheel with the same discretisation level and the same radius but with a tenfold thickness. Such a structure is similar to a cube.

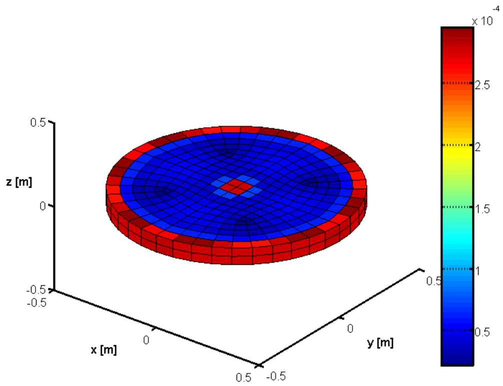

Fig. 3: Relative error of the boundary elements of the structure at a frequency of 500Hz.

In Fig. 4 the surface and field error for both wheels are presented. The surface error depends substantially on the geometry of the radiating structure, whereas the radiation error hardly differs concerning the geometrical structure. The irregularity of the error curves for the thick wheel at around 300 Hz is due to an eigenfrequency of the associated interior Dirichlet problem, which occur at this frequency. This eigenfrequency was treated by using five chief points around the centre of the structure.

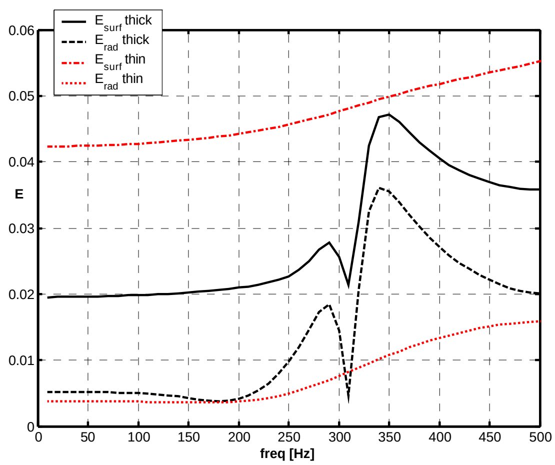

Fig. 4: Surface and radiation error for a thin (radius = 0.5m, thickness = 0.1m) and a very thick wheel (radius = 0.5m, thickness = 1m).

It can be subsumed, that the BEM program yields a good approximation of the radiated sound pressure according to the monopole source test. On the other hand, the monopole source represents hardly the more complex vibration modes of wheel-like structures. Therefore it should be investigated, whether sources of higher order are more suited for the test procedure.

SOUND RADIATION

The sound radiation was calculated for a wheel model of the following properties: radius = 0.5 m, and thickness = 0.1m, consisting of steel with a Young's modulus $E=2.1 \cdot 10^{11}$ N/m^2, density ρ = $7.8 \cdot 10^3$ kg/m^3 and a Poisson' ratio of 0.3. A free-free boundary condition for the support of the wheel model was chosen.
In order to solve the Helmholtz integral equation for a Neumann boundary condition, the normal velocity v_n on the surface has to be known. To get these velocity data for different eigenfrequencies

a modal analysis of a FEM-model of the structure was done using ANSYS. The data of the structural modes were then transferred to the BEM program. Fig. 5 shows the first two modes shapes of the wheel, as they were calculated with ANSYS and Fig. 6 the radiation directivity of these mode shapes. The characteristic radiation directivity correspond very well to the respective mode of vibration.

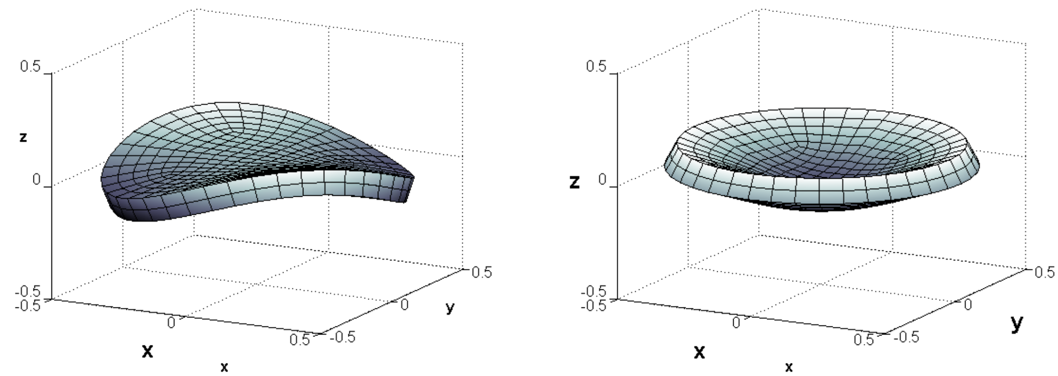

Fig. 5 : left: first mode shape of the wheel like structure at a frequency of 512 Hz, **right:** second mode shape at 852 Hz

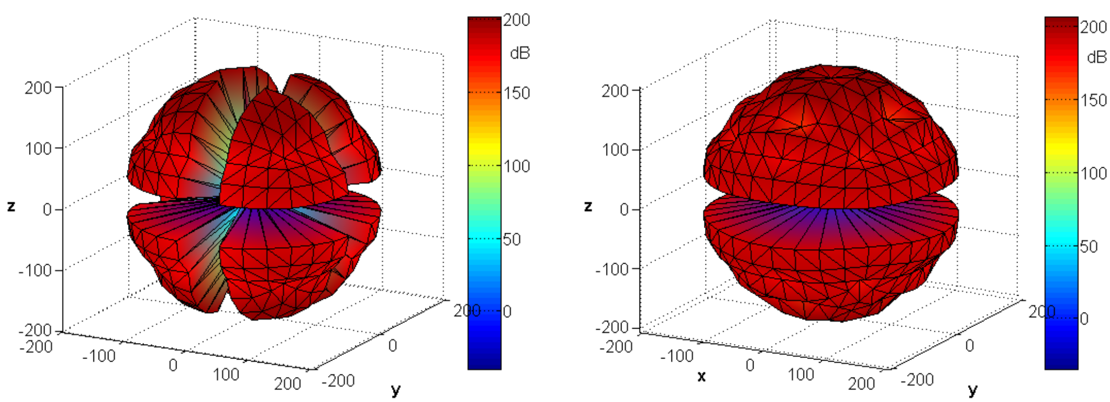

Fig. 6: left: 3D-directivity of the first eigenfrequency at 512 Hz **right:** 3D-radiation directivity of the second eigenfrequency at 852 Hz

In the first step only the sound radiation of the eigenfrequencies of the structure was calculated. The calculation of a frequency response of the radiated sound power, based on a modal superposition, is planned for the future.

CONCLUSIONS AND OUTLOOK

On the basis of the a test procedure it could be shown, that the developed BEM algorithm yields a good solution for the calculation of the sound radiation from wheel-like structures. First results could be presented, which show the radiation directivity of the investigated structures for selected modes of vibration. Nevertheless, there are several steps planned to improve the program, such as the implementation of surface elements with a higher shape function than constant elements or the improvement of the integration algorithm. Also, the treatment of the characteristic eigenfrequencies, which cause a non-unique solution of the exterior Helmholtz integral equation will be changed from the so far used CHIEF method to the Burton and Miller method, [8]. Considering the computationally intensive problems, which emerge in 3-D acoustics, it will be also necessary to implement iterative solvers for the set of linear equations as done in [4]. However, the main point will be the extension of the model to the case of a ground impedance and a noise barrier. There are various publications, that deal with the sound propagation above an impedance surface, e.g. [1], [3], [5], and the influence of a sound barriers, [2]. These will be taken into account to derive an appropriate Green's function for the sound propagation above a barrier on flat ground. For a verification of the program it is planned to compare the results with results from the BEM program LMS SYSNOISE.

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