# ACOUSTIC SYSTEM SIMULATION USING THE TRANSMISSION LINE MATRIX METHOD

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**Abstract**

The Transmission Line Matrix method (TLM) provides an easy to use and fast scheme to calculate acoustic fields. Thus it is possible to couple this method to system simulation tools in order to investigate the behavior of acoustic systems and the interaction between acoustic domain and control electronics at an early design stage. These investigations are very important especially in design of micro electromechanical systems (MEMS), such as ultrasonic devices, transducers and SAW filters. As a proof of concept an active noise control system was designed and the performance was simulated in a 2D acoustic environment. The results clearly show the feasibility of acoustic field calculations along with system level simulations of electronic devices and transducers. As another test case, the TLM model was applied to a two dimensional section through an ultrasound transducer array to study surface waves. Despite the simplicity of the deployed models the simulation exhibits the relevant features of surface wave generation and propagation. Using this scheme acoustic devices may be simulated and optimized under realistic acoustic conditions.

## 1 Introduction

The ongoing miniaturization of micromachined acoustic systems on the basis of highly sophisticated production processes results in extremely high prototyping costs at the design stage of a system. Therefore a system level simulation is necessary to optimize and verify the system performance. For electronic and several mechanical components tools like SPICE, Saber or VHDL-AMS simulators provide efficient means for such system level simulations. However, in many cases important effects occur in the acoustic domain which can not be described by simple equations. For system level simulations of acoustic devices a method for the rapid solution of the acoustic domain problems is needed which may be coupled to appropriate simulation tools easily. For wave propagation problems the Transmission Line Matrix (TLM) method is established as a simple and very efficient algorithm for the solution of 2 and 3 dimensional problems. For that reason it is well suited for the coupling with system level simulators.

The TLM method has been applied successfully to many problems which are described by wave or diffusion equations. The main difference between the TLM approach and standard schemes such as Finite Difference or Finite Element methods is as follows: Usually a physical problem is described precisely by a partial differential equation which subsequently is discretized. The solution to this equation is then approximated numerically. The TLM approach is to approximate the physical process (e.g. wave propagation) by discrete physical models, which can then be calculated exactly. This approach has several advantages. The physical models may be programmed intuitively, all discretization approximations are apparent, and the calculation may be parallelized easily by domain decomposition. Furthermore the coupling to system simulation tools is well possible as the field is discretized in lumped elements which nicely fits into the philosophy of the simulation of discrete elements used in system simulators. The disadvantages are that TLM is best suited to equidistant nodes and fixed time steps, which are coupled to the spatial distance of the nodes. However, in recent years TLM was adapted for non-equidistant and curvilinear grids.

In this study the TLM scheme is coupled with the system simulation tool SABER. As a very descriptive example an active noise control system is simulated, which consists of microphones and a loudspeaker situated in an air filled pipe. Sound generated by an artificial source enters the pipe from the left side. The aim is to reduce the sound pressure level at the exit. Here little effort was made to construct an efficient active noise control system, instead the performance of a given system was to be evaluated.

As a further test system an array of capacitive micromachined ultrasound transducers was investigated. The TLM method was used to simulate the generation and propagation of surface waves on the array, which a results of the acoustic coupling of the single transducers. These surfaces occur at locally reacting compliant walls. If the transducer array is used for transmission and reception such as in e.g. in medical diagnostics, these surface waves are unwanted because of their long ring down times and their ability to disturb the measurement of the echo signals.

## 2 Theory

The TLM scheme may be introduced using an approach by Kagawa which relies on a physical model for the acoustic field [1, 2]. The basic idea is to follow Huygens principle that states, that each point on a wavefront can be considered as a source of secondary wavelets, so that the propagation can be considered as a continuous process of scattering of elementary component waves. The superposition of all scattered (elementary) waves makes up the wave. This mechanism may be discretized as follows: The fluid is replaced by a network of pipes with equal diameters which are joint at fixed distances $\Delta x$ (see figure 1). The junctions are called nodes. Within the pipes acoustic pressure pulses may travel undisturbed at the speed of sound, i.e. no damping or boundary effects are considered. The diameter of the pipes is assumed to be much smaller than the shortest wavelength of interest so higher order pipe modes are neglected. From this follows, that if a set of acoustic pressure pulses is released at the same time at arbitrary nodes, these pulses travel through the pipes and reach their neighboring nodes after a time interval of $\Delta t = \Delta x / c_0$ with $c_0$ being the speed of sound in the pipe. For 2 dimensions, the junction may be seen as a meeting of the pipe carrying the initial, incoming pressure pulse with three outgoing pipes. Thus the acoustic pressure pulse arriving at the node sees an apparent enlargement of the cross section by a factor of three, which means a corresponding change of the impedance. At this impedance mismatch the incoming pressure pulse is reflected with half of the initial amplitude and a reversal of sign, while it is transmitted into the other three pipes with half the initial amplitude and no change of sign (see figure 1). From this point on the process of traveling in the pipes and scattering at the nodes is repeated, with four pulses arriving simultaneously at each node and each scattering into the other three pipes. As the process of scattering and traveling is linear, superposition applies and the net result is the sum of the component effects. The algorithm described above can formally written as follows:

$$f_i(\vec{x}+\Delta\vec{x}_i, t+\Delta t) = S_{ij}\, f_i(\vec{x}, t) \tag{1}$$

with $f_i(\vec{x}+\Delta\vec{x}_i, t+\Delta t)$ representing the amplitude of the pressure pulse that travels from the node at position $\vec{x}$ to the node at position $\vec{x}+\Delta\vec{x}_i$ between times $t$ and $t+\Delta t$. $\Delta\vec{x}_i$ is the vector pointing to the nearest neighbor in the $i$-th direction, and $S_{ij}$ is the scattering matrix

$$S_{ij} = \frac{1}{2}\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

Because of the simple structure of the scattering matrix equation (1) may also be written as

$$f_{i^*}(\vec{x}+\Delta\vec{x}_i, t+\Delta t) = \frac{1}{2}\sum_j f_j(\vec{x}, t) - f_i(\vec{x}, t) \tag{2}$$

where $i^*$ is defined such that $\vec{x}_i = -\vec{x}_{i^*}$. The acoustic pressure $p_{ak}$ and the velocity $\vec{v}$ are calculated from $f_i(\vec{x}, t)$ applying the following formulae:

$$p_{ak}(\vec{x}, t) \quad = \quad \frac{1}{2}\sum_i f_i(\vec{x}, t) \tag{3}$$

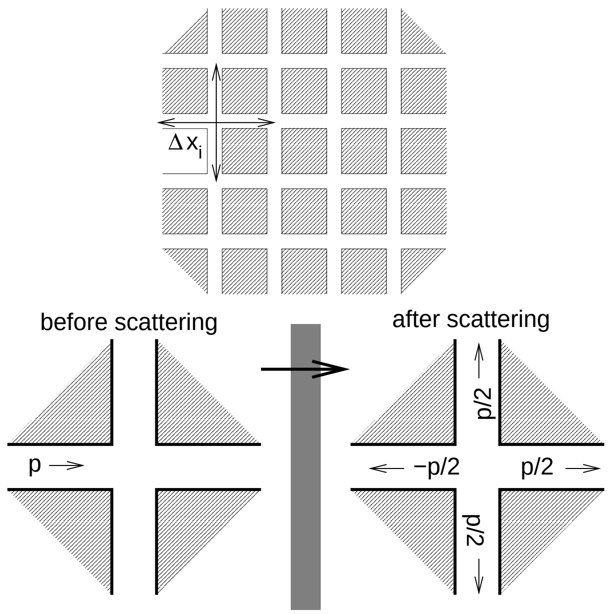


Figure 1: The acoustic medium is replaced by a network of pipes, in which pressure waves travel. The junctions are called nodes, the vectors $\Delta\vec{x}_i$ point from one node to the nearest neighbors. The scattering process is depicted in the lower part of the figure.

$$v_{x,y}(\vec{x}-\frac{\Delta\vec{x}_i}{2},t-\frac{\Delta t}{2}) \quad = \quad \frac{1}{\rho c^2}\frac{\Delta\vec{x}}{\Delta t}\left(f_i(\vec{x}i-\Delta\vec{x}_i,t)-f_{i^*}(\vec{x},t)\right) \tag{4}$$

if the $\Delta\vec{x}_i$ are parallel to the axes of the co-ordinate system. This scheme is numerically equivalent to a second order Finite Difference scheme for the wave equation, which is shown as follows: The acoustic pressure at a position $\vec{x}$ and time $t+\Delta t$ is given as

$$p_{ak}(\vec{x},t+\Delta t)=\frac{1}{2}\sum_i f_i(\vec{x},t+\Delta t)$$

Repeated use of eqs.(2, 3) yields

$$\begin{aligned} p_{ak}(\vec{x},t+\Delta t) &= \frac{1}{2}\sum_i p_{ak}(\vec{x}+\Delta\vec{x}_i,t)-f_{i^*}(\vec{x}+\Delta\vec{x}_i,t) \\ &= \frac{1}{2}\sum_i p_{ak}(\vec{x}+\Delta\vec{x}_i,t)-p_{ak}(\vec{x},t-\Delta t)+f_i(\vec{x},t-\Delta t) \end{aligned}$$

This may be rearranged to give

$$p_{ak}(\vec{x},t+\Delta t)-2p_{ak}(\vec{x},t)+p_{ak}(\vec{x},t-\Delta t)=\frac{1}{2}\sum_i p_{ak}(\vec{x}+\Delta\vec{x}_i,t)-2p_{ak}(\vec{x},t)$$

which equivalent to the corresponding Finite Difference equation if

$$\frac{\Delta x}{\Delta t}=\sqrt{2}c$$

This last result means that the speed of wave propagation $c$ is given by the ratio of the spatial and temporal resolution. Note that the overall wave speed $c$ is lower by the factor $1/\sqrt{2}$ than the speed of component pressure pulses traveling in the pipes, which is a feature of the method that is surprising when first encountered.

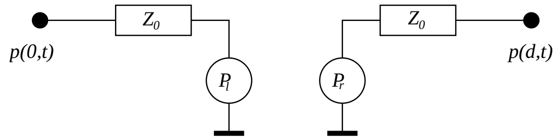

Figure 2: Equivalent circuit model for the lossless transmission line, which corresponds to the pipes in the TLM mesh.

## 3 Coupling Saber and TLM

From a theoretical point of view the TLM scheme is well suited for the combination with a system simulator, because an equivalent circuit model exists, which makes the formulation of coupling equations quite easy. The model for the lossless transmission line following Branin [3] is

$$\begin{aligned} Z_0 u(0,t) &= P_r(t+\tau) \quad -p(0,t) \\ Z_0 u(d,t) &= \quad -P_l(t) \quad +p(d,t+\tau) \end{aligned}$$

where $u(x,t)$ is the velocity of the fluid, $Z_0$ is the acoustic impedance and $p(x,t)$ is the pressure at the corresponding ends of the transmission line at time $t$. $P_l(t)$ and $P_r(t)$ are pressure sources as shown in fig. 2 and correspond to the amplitudes of the pressure pulses given by $f$ in the preceding section. The pressures generated by the sources are updated at each time step using

$$\begin{aligned} P_r(t+\tau) &= 2p(0,t) - P_l(t) \\ P_l(t+\tau) &= 2p(d,t) - P_r(t) \end{aligned}$$

Using this equations one is free to choose either the pressure or the velocity as input while the TLM model calculates the complementary values as output. Here, the velocity of the membrane of the loudspeaker was used as input. The resulting force on the membrane of the speaker was calculated as the integral of pressure on the membrane area, which in turn was computed by

$$p(x,t) = v(x,t)Z_{ak} + f(x+\Delta x, t-\tau)$$

For the coupling of the system simulator SABER to the TLM model the C interface of the behavioral description language MAST was used. The TLM code was embedded into a MAST model and is called as a foreign routine at equidistant time steps. The step width is equal to the internal TLM time step. To ensure the equidistant call of the TLM model an internal clock signal was created using the MAST statements for event driven models.

## 4 Results

To evaluate the coupled simulation a well known problem from acoustics - active noise control - was chosen. The test setup consists of a pipe that might be the exhaust of some industrial machinery where sound enters through the left end. The aim is to reduce the amplitude of the sound that leaves the pipe at its right end by generating a complementary sound signal in the middle of the pipe with a suitable loudspeaker. Clearly the performance of this active noise control system is strongly dependent on the geometrical properties of the acoustic environment (dimensions of the pipe and the loudspeaker) as well as mechanical and electrical properties of the transducers and the design of the controlling system that generates the signal applied to the loudspeaker. In this simulation only the influences of the acoustic environment are taken into account. The physical sizes and parameter of the acoustic calculation are given in table 1.

For active noise control the pressure signal from microphone is delayed and low pass filtered. Then it is amplified and applied to the speaker, where the velocity of the membrane is taken to be proportional to the electrical signal (ideal loud-speaker). The electrical signal of the loudspeaker is delayed and subtracted from the original microphone signal. This corrected microphone signal is used as input for the forward delay block. For the given setup delaying and filtering was implemented using control blocks from SABER. The next step in design would be to use electrical circuitry for the active noise control.

The special properties of wave propagation in TLM meshes are well described in literature [4, 2, 5] and therefore were not subject to tests in this study.

| | |
|---|---|
| length of pipe | 2 m |
| diameter | 0.2 m |
| loudspeaker membrane diameter | 0.2 m |
| distance of microphone from pipe end | 0.2m |
| speed of sound | 340 m/s |
| spatial resolution | 2 mm |
| time step | 4.2 $\mu$s |
| fluid density | 1.21 kg/m$^3$ |
| number of TLM nodes | 1000x100 |

Table 1: Physical parameters of the test setup and the acoustic calculation.

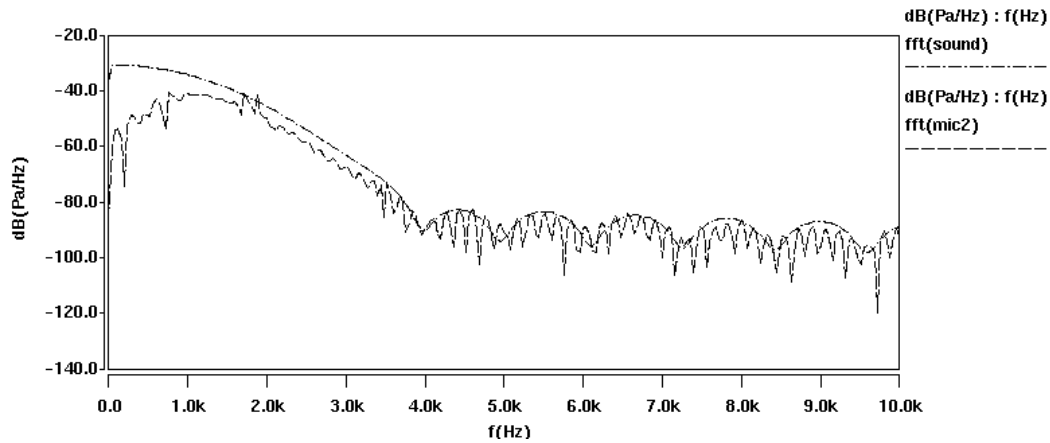


Figure 3: Power spectral density of Gaussian shaped input impulse and the sound pressure at the exit of the pipe.

To evaluate the method several calculations were performed, where a sound wave was fed into the left end of pipe. The pressure signal calculated at the microphone position was passed to Saber. Inside Saber the controlling system was simulated, which resulted in a calculated velocity of the loudspeaker membrane. This velocity was given to the TLM subroutine which controlled the boundary conditions at the position of the loudspeaker membrane accordingly. No efforts were made to apply more realistic models of the transducers to account for frequency response, membrane weights etc. in order to keep the example simple. However, for further design steps more realistic models of the transducers could be applied easily.

For this simple test case only the time delays and the amplification of the microphone signal had to be adjusted. Furthermore, to prevent oscillations the parameters of the low pass had to be set up.

In order to get sufficient input information a Gaussian shaped pressure pulse was released at the left end of the pipe and the system evolution was calculated for the following 0.02 s. The system performance was monitored by comparison of the power spectral densities of the input pressure pulse to that of the signal at the exit of the pipe (fig. 3).

Significant damping values can be observed below 1 kHz, the maximum damping is about 30 dB.

The calculation of surface waves on a compliant wall requires the surface to be modeled. Here, a very simple approach was used: At each TLM node adjacent to the compliant wall a mass-spring system was used to simulate the local behavior of the wall. The compliance of the spring and the mass were adjusted according to the given values for the membranes of an ultrasound transducer array. The mass spring system was described with an ordinary differential equation which was solved numerically using a semi-implicit Newmark scheme. Fig. 4 shows a snapshot of the pressure field. After 10% of the membrane on the left were excited a sound wave was released into the fluid, while at surface additional pressure patterns show up, which can be related to surface waves. Further inspection revealed that the surface waves observed in the TLM model satisfy the appropriate dispersion relation. A reference calculation using a FEM-model was in good agreement with the results shown here.

Figure 4: Snapshot of the pressure field over a compliant wall. The wall is at the bottom boundary, at the top and right absorbing boundary conditions were applied while the left boundary condition was set to ideal reflection. Grey means average pressure while white and black corresponds to areas of higher/lower pressure, respectively. The black and white patterns along the bottom boundary belong to surface waves, which decay exponentially away from the surface.

## 5 Conclusions

The goal was to establish a computational framework that allows to simulate complex devices with acoustic subsystems and electronics. This was achieved by coupling the system simulation tools SABER with the TLM method, which was implemented as subroutine in C and linked to SABER.

For demonstration of the approach system simulations of an active noise control system in a pipe were carried out. These simulations served to support the design of the structure of the active noise control unit, to define the parameters of the blocks and to verify the performance of the system. Although most of the computation time was consumed by TLM, due to its high efficiency system simulation were possible in a reasonable time (about 400s computing time for 100ms real time on a SUN Blade1000, 750MHz CPU, 1GB RAM).

Future applications of the coupled tools is the design of micromachined transducer arrays, where complex problems of parasitic surface waves shall be studied in 3D. First 2D test calculations clearly show the ability of the system to reproduce the system behavior with respect to surface waves.

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