# The Application of Boundary Integral Equation to Wide Barriers in Turbulent Medium

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## ABSTARCT

Turbulence is an important factor that limits the performance of a noise barrier. Current numerical models for predicting the effect of turbulence on a barrier include the Substitute Sources approach, which has been shown to be fast and effective for a single vertical barrier. However for a wide barrier the Rayleigh Integral is not exact and the Boundary Integral Equation (BIE) is more appropriate. In this paper the effect of de-correlation of the Green's function in the BIE due to turbulence is studied. Results are compared with prediction by other approaches and measured data on a two barriers.

## INTRODUCTION

The use of barriers to shield sensitive areas from noise sources is a common approach in traffic and environmental noise control. Over the years there have been numerous studies into enhancing noise barrier performance by means of special shapes and surface treatments. The boundary integral equation (BIE) method [1] has proved to be an effective way of predicting the performance of these innovative barrier designs [2]. It can readily deal with complicated barrier shapes and surface conditions and the formulation is exact and the results are highly accurate. With the continuous reduction of desktop computing power the BIE method has become ever more available to a wide range of practical problems.

In practice, the performance of noise barriers is limited by many factors. One of which is atmospheric turbulence. It has been known for a long time that turbulence restricted the amount of transmission loss that can be achieved by a barrier and a maximum value of 20dB for a single barrier is commonly used. The estimation of reduction in barrier performance due to turbulence is particularly important for innovative barriers where the interaction of the shape and turbulence can be significant.

In recent years a number of ways of simulating the effect of isotropic, homogeneous turbulence on barrier noise insulation has been used. They may be considered in three broad categories – i) modelling the energy scattered into the barrier shadow zone by the scattering cross section of the turbulence structure [3,4], ii) simulating the turbulence effect by random perturbations of the sound speed profile [5,6], and iii) incorporating the turbulence effect as a de-correlation of sources by means of the mutual coherence function (MCF) [7,8]. Methods in category (i) approximate the turbulence scattering as a separate phenomenon from the barrier diffraction. Methods in category (ii) require a suitable numerical computation scheme such as the Parabolic Equation (PE) that can include the perturbed sound speed profile and are generally computationally expensive. An example of category (iii) methods is the Substituted Source Method (SSM) [9] in which the source and barrier is replaced by a plane of substituted sources at the barrier by applying the Rayleigh Integral. The sound pressure at the receiver is then calculated from the turbulence de-correlated contributions from the sources. The SSM is simple to implement and has a lot of potential for practical applications. However, the Rayleigh Integral formulation is exact only if the strength of the substituted sources are determined exactly, with the inclusion of the barrier diffraction effect. For barriers of complicated shapes this would require advanced barrier diffraction models such as the BIE method in the first place. Ideally it would be useful if the source de-correlation concept can be incorporated into the diffraction model directly rather than creating a separate substituted source plane. This could also eliminate the problem of having to deal with the infinite extent of the substituted source plan. This paper investigates the feasibility of applying the source de-correlation concept to the calculation of barrier attenuation in a turbulent atmosphere using the BIE method.

## THEORY

The formulation of the BIE for an exterior diffraction problem may be written as [1,2]:

$$\int_S p_s(r_o)\frac{\partial G(r\,|\,r_o)}{\partial n_o} - G(r\,|\,r_o)\frac{\partial p_s(r_o)}{\partial n_o}\,ds = \varepsilon p_s(r) \qquad (1)$$

where $p_s$ is the scattered pressure component and $G$ is the Green's function for the free space problem. The coordinate vectors are $\boldsymbol{r}$ and $\boldsymbol{r_o}$ where the subscript $o$ indicates that the position is on the surface of the barrier $S$. $\boldsymbol{n_o}$ is outward normal at $\boldsymbol{r_o}$. The constant $\varepsilon$ takes the values of 1, ½ and 0 if $\boldsymbol{r}$ is respectively in the exterior region $V$, on the boundary $S$, or in the interior of the barrier $V_{in}$.

Note that the formulation is on the scattered pressure $p_s$. The total pressure $p$ at $\boldsymbol{r}$ is given by

$$p(r) = p_s(r) + p_{inc}(r) \qquad (2)$$

where $p_{inc}$ is the incidence pressure from the source. For convenience, since most physical problems are defined with boundary conditions specified on the total rather then the scattered pressure, one may modified Equation (1) by first noting that,

$$\int_S p_{inc}(r_o)\frac{\partial G(r\,|\,r_o)}{\partial n_o} - G(r\,|\,r_o)\frac{\partial p_{inc}(r_o)}{\partial n_o}\,ds = 0, -\frac{1}{2}p_{inc}(r), -p_{inc}(r) \quad for\ r \in V, S, V_{in} \qquad (3)$$

Adding Equations (1) and (3) together results in the more common formulation for the total pressure $p$ in $V$,

$$\int_S p(r_o)\frac{\partial G(r\,|\,r_o)}{\partial n_o} - G(r\,|\,r_o)\frac{\partial p(r_o)}{\partial n_o}\,ds \;+\; p_{inc}(r) = \varepsilon p(r) \qquad (4)$$

An exact solution of the BIE can be obtained if one can determine a Green's function for the turbulent medium, say for example by solving an integral over the turbulent domain. However this is likely to be rather computationally expensive.

The terms inside the integral may be considered as representing a distribution of point sources (involving $G$) and dipole sources (involving $G$). Hence the simple concept of source de-correlation by the turbulence atmosphere can be applied to these sources. As a first approximation, the strength of these surfaces sources may be determined by solving the BIE (Equation(4)) for a stationary, non-turbulent medium. This is similar to the assumption used for the determination of the source strength in the SSM [9], and should be acceptable if the actual source is close to the barrier so that the source-to-barrier propagation is not much affected by the turbulence.

Representing the contribution from each of these sources by $Q_i$ where $i=o$ is the incident term, the long-term average of the square of the pressure amplitude can be written as:

$$< p^2 > = \sum_{i=o}^{N} | Q_i |^2 + 2\sum_{i=0}^{N-1} \sum_{j=i+1}^{N} | Q_i Q_j | \cos\left[ \arg\left( \frac{Q_j}{Q_i} \right) \right] \Gamma_{ij} \qquad (5)$$

where $\tilde{A}_{ij}$ is the MCF between the sources *i* and *j*. $\tilde{A}_{ij}$ for spherical waves has been derived for Gaussian and Kolmogorov turbulence spectra. Note that the surface sources in Equation (4) include both point and dipole sources. In here it is assumed that the MCF for point sources can be used for dipole sources. The MCF for a Gaussian turbulence model is given by [7,8],

$$\Gamma( r, L) = e^{-\sqrt{p}\, m_o{}^2 k^2 \ell L \left( 1 - \frac{f( r / \ell)}{r / \ell} \right)} \qquad (6)$$

where *k* is the wavenumber of the non-perturbed medium, $ì_o$ and      are the standard deviation of the fluctuating part of the index of refraction and the correlation length of the Gaussian spectrum, *ñ* and *L* are the transversal and Longitudinal distances for two sources and one receiver, and the function

$$f(x) = \int_0^x e^{-u^2}\, du$$

By de-correlating between the scattered sources and the incident source, the cancellation of $p_s$ and $p_{inc}$ that gives rise to the small sound pressure in the shadow zone of the barrier will be reduced and the total pressure *p* will be increased.

## RESULTS

The first test case chosen is the 10m thin barrier used in Forssén's [6] comparison of PE and scattering cross-section modelling of the effect of turbulence on barrier noise reduction. The source and receivers are on a hard ground. The SPL at receivers from 100m to 1000m away from the barrier were calculated. Figure 1 shows the result for a close-by source - at 1 m from the barrier. BIE calculations were performed for no turbulence, with the MCF between sources calculated with a typical    =1.1 and two values of $ì_o{}^2$ $=3\text{x}10^{-6}$ (moderate) and $3\text{x}10^{-5}$ (strong). Also shown is the result for the extreme case of total de-correlation (all $\tilde{A}_{ij}$=0). The predicted effect of the turbulence for the two turbulence cases appears as expected. The barrier transmission loss is reduced slightly with $ì_o{}^2$ $=3\text{x}10^{-6}$ and significantly with $ì_o{}^2$ $=3\text{x}10^{-5}$. The effect increases with range as the MCF is reduced exponentially with *L* (Equation (6)).

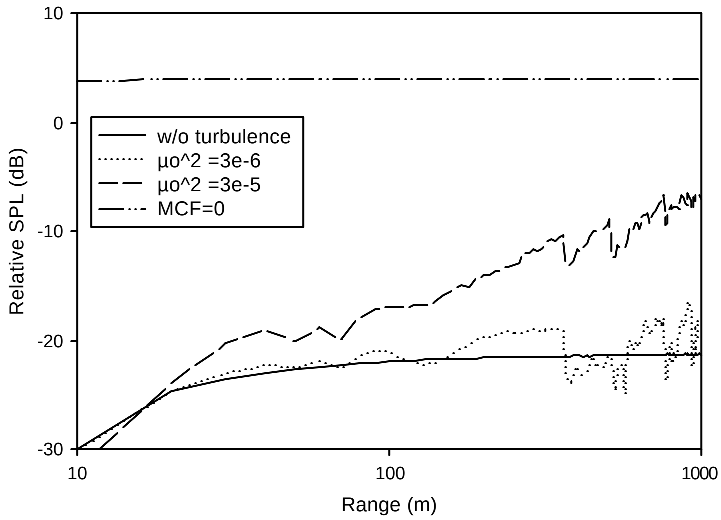


**Figure 1** BIE prediction at 1000Hz for a 10m high thin barrier. Source at 1m.

The predicted result for the extreme case is however less satisfactory. When all $\tilde{A}_{ij}$=0 the scattered sources are incoherent with each other. The predicted relative sound level approaches a constant value of about

+4dB at all distances > 50m, meaning that the sound level in the presence of the barrier, under this extreme case, is higher than that without the barrier. An indication of the cause of this over-prediction can be seen by examining the BIE formulation, Equations (1) to (4). In the shadow zone of the barrier, the small relative sound level is created by the cancellation of the incidence pressure $p_{inc}$ and the scattered pressure $p_s$. Hence the absolute amplitude of $p_s$ is of the same order of magnitude as $p_{inc}$. When the sources become incoherent, the cancellation does not occur and the overall level is given by the sum of the incoherent energy from each sources. Hence the level will be higher than that of $p_{inc}$ on its own. This is however likely to be a problem of de-correlating the BIE formulation rather than a physical occurrence.

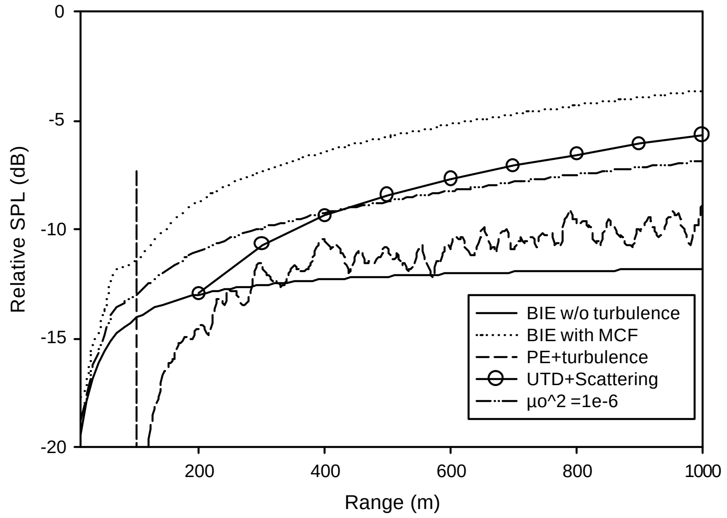


**Figure 2** Comparison of various predictions at 500Hz for a 10m high thin barrier. Source at 100m.

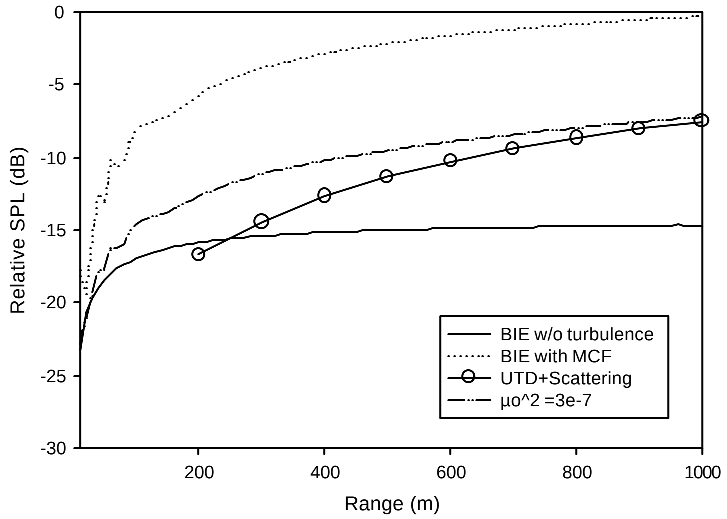


**Figure 3** Comparison of various predictions at 1000Hz for a 10m high thin barrier. Source at 100m.

The tendency of the de-correlated BIE to over-predict the effect of turbulence can be clearly seen in Figure 2 when the prediction is compared with the alternative predictions by the PE simulation and by the method of scattering cross-section for a case when the source is 100m from the barrier. The data for the later two predictions were taken directly from Forssén’s [6] and UTD refers to the uniform theory of diffraction. The calculations were all done for typical Gaussian turbulence parameters of =1.1 and $ì_o^2$ $=3x10^{-6}$. The BIE+MCF prediction is about 2-3 dB higher than that by the method of scattering cross-section. Only when the parameter $ì_o^2$ is reduced to $1x10^{-6}$ then the BIE+MCF prediction becomes close to the scattering cross-section prediction. As can be seen in the BIE formulation, the coherence between the surface sources plays an important part in representing the physical condition of the barrier (such as maintaining $p(\mathbf{r})=0$ when $\mathbf{r}$ is in the interior of the barrier $V_{in}$). Hence the de-correlation of the surface sources not only creates a turbulence scattering effect but may also affect the integrity of the BIE formulation itself. Physically one may consider that, with the surface sources de-correlated, some sound energy can now pass through the barrier itself to reach the receiver (since $p(\mathbf{r})$ is no longer $0$ when $\mathbf{r}$ is in the interior of the barrier $V_{in}$), as well as scattered by the turbulent medium. The overall effect is therefore an over-prediction of the sound level at the receiver. This error is larger at higher frequencies because of the $k^2$ dependence in the exponent of the MCF (see Equation (6)). Figure 3 shows the case for 1000Hz. The over-prediction, in comparison with the scattering cross-section result, is now about 8 dB when the same $ì_o^2$ $=3x10^{-6}$ is used. Better agreement is achieved only by reducing $ì_o^2$ to $3x10^{-7}$.

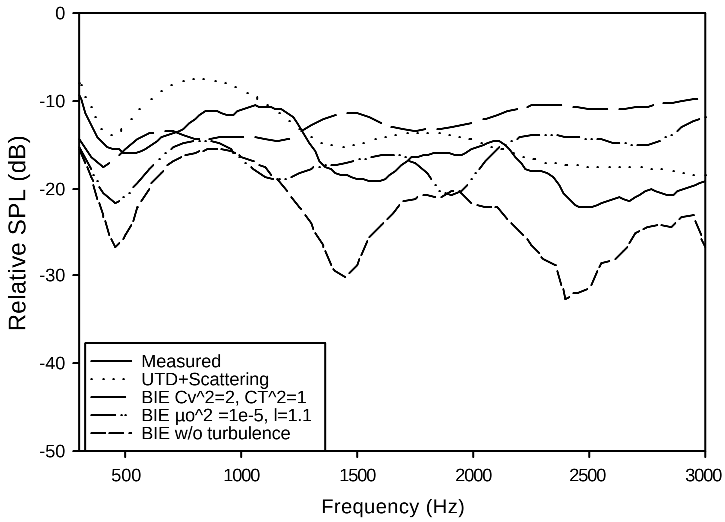


**Figure 4** Comparisons for a thick barrier. Source at 8m. Receiver at 18m. Measured $C_T^2=1$ and $C_v^2=2$.

A problem with the comparisons shown in Figures 2 and 3 is that the source is very far from the barrier (100m). One of the assumptions of the current BIE+MCF method is that the surface pressure can be approximately calculated using the non-turbulent conditions. Significant error is therefore expected when the source is far away and there is a significant distance for the sound to propagate through the turbulent medium to the barrier. It should also be pointed out that the MCF equation used here is derived for spherical waves, while the surface sources in the BIE formulation are all dipole sources when the barrier is rigid ( $p/\ n_o=0$ in Equation (4)). This could also be a significant source of error.

A better case to check the validity of the current method is the measurement on a thick barrier described in Ref.[10]. The barrier in this case is 2.55m high and has a thickness of 2.44m. The source is 8m away from the nearest face of the barrier and so is not too far away. The measurements were conducted at 3 distances with the furthest at 18m from the face of the barrier. The atmospheric conditions were also measured and were used to derive the parameters $C_T^2$ and $C_v^2$ for a Kolmogorov turbulence model. The MCF derived under this model is given by,

$$\Gamma(\mathbf{r},L) = e^{-\frac{3}{8}D\left(\frac{C_T^2}{T_o^2}+\frac{22}{3}\frac{C_v^2}{c_o^2}\right)k^2 r^{\frac{5}{3}}L} \qquad (7)$$

where $T_o$ is the mean temperature in Kelvin, $c_o$ is the mean velocity of sound, and $D$ is approximately 0.364.

Figure 4 shows a comparison for a case when the turbulence is moderate, with the measured $C_T^2$=1 and $C_v^2$=2. Also shown is the BIE calculation using MCF calculated by the Gaussian model using parameters =1.1 and ì$_o^2$ =1x10$^{-5}$. The UTD + Scattering cross-section data were taken from Ref.[10]. Over the frequency range of 250Hz to 2kHz the prediction by the BIE with the Kolmogorv MCF agrees quite well with the measured data and the scattering cross-section prediction. The BIE with the Gaussian MCF also gives a reasonable prediction. At higher frequencies the tendency of the BIE+MCF predictions to over-predict the sound level becomes apparent.

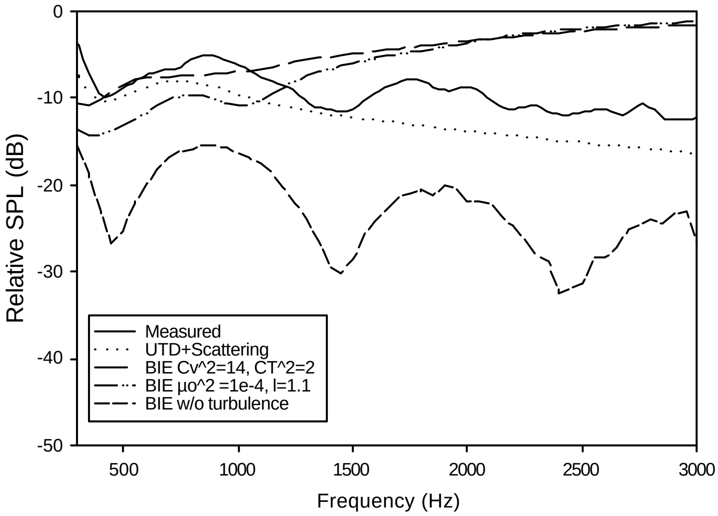


**Figure 5** Comparisons for a thick barrier. Source at 8m. Receiver at 18m. Measured $C_T^2$=2 and $C_v^2$=14.

Figure 5 is for a strong turbulent case, with the measured $C_T^2$=2 and $C_v^2$=14. Again, prediction by the BIE with MCF calculated by a Gaussian model using =1.1 and ì$_o^2$ =1x10$^{-4}$ is also shown. The result is similar to that observed in Figure 4, except that the very strong turbulence parameters resulting in a much faster de-correlation of the BIE formulation with the Kolmogorov MCF. The weaker parameters used in the Gaussain MCF gives a lesser effect.

## CONCLUSIONS

The combination of the BIE with the MCF is a convenient way of calculating the performance of a noise barrier under turbulent atmospheric conditions. It has been shown in this paper that the prediction agrees well with measurements and predictions by the scattering cross-section method when the de-correlation, i.e. the perturbation due to turbulence, is small. In this paper this condition corresponds to cases when the source is close to the barrier (around 8m) and low to mid frequencies (< 2000Hz). At higher frequencies and at longer source distances the method tends to over-predict the effect of turbulence. This is caused by the strong dependency of the BIE formulation on the coherent cancellation of the scattered and incident sound sources to represent the physical blocking effect of the barrier. When the coherence is artificially reduced the physical integrity of the formulation is also reduced and over-prediction of the turbulence effect occurs. This is demonstrated in a test case where the source is far (100m) from the barrier. In this case the prediction by the BIE+MCF on the reduction of barrier attenuation is about 4dB higher than that by the scattering cross-section method at 500Hz, rising to about 8dB at 1000Hz.

An alternative way of modelling the effect of turbulence in the BIE formulation is to use the use the turbulence parameters to generate random realisations of the amplitude and phase perturbations on the sources directly. One may also use the *kr* scaling property to represent the perturbations by small variations of the barrier geometry – similar to random roughness. This approach will allow the physical integrity of the BIE formulation to be maintained and therefore may not suffer from the same error as the MCF approach. Further work will need to be done to verify this idea.

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