THE EQUIVALENT SOURCES METHOD APPLIED TO A 2D CITY CANYON

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Abstract

A common situation in an urban traffic environment is straight roads with high buildings on both sides, forming a city canyon. This problem can be simplified into a two dimensional situation where the road traffic is included as a line source. The problem can be divided into two domains, a room-like cavity and a free field above it. The two domains are then coupled by a set of sources. The model can be extended to a situation where one or more sources are located in one canyon and the receiver is located in an adjacent canyon.

1 INTRODUCTION

The sound level on the noisy side of a building in a city environment , including reflections from nearby façades, can be predicted with many standardized methods, for example [1]. The level at shielded courtyards is however more difficult to predict. Ray based methods are difficult to apply, and often give too low levels at such positions [2]. Therefore some effort have been put into finding a better method for such positions. This paper presents a different approach based on the method of equivalent sources, which might be more reliable.

The approach of using equivalent sources to couple two domains have been used for similar problems earlier by e.g. Kropp [3] and Cummings [5]. However, due to the size of the geometries considered here in 3D it could only be used for low frequencies due to the computational effort. Using a 2D model is therefore a natural first step in order to deal with problems where the dimensions (5–20 m) correspond to many wavelengths at the most important frequencies (500 Hz–1kHz).

The geometry considered in this paper is outlined in figure 1, where Γ denotes the boundary between the canyon and the free half-space above it.

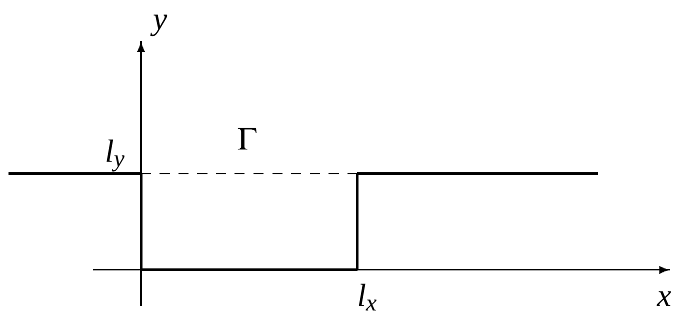

Figure 1: Coordinate system of the 2D city canyon.

2 THEORY

2.1 The Green functions

The main idea of the method of euqivalent source is to describe the unknown impedance at the interface between canyon and halfspace (Γ) by a set of monopoles which are adjusted in such a way that continuity of the sound field at the interface is mantained. This demands so-called Green functions for the radiation from the interface into the halfspace and into the canyon, which in this case can be considered as a cavity with rigid walls.

The following assumes a harmonic time dependence described by $e^{\mathrm{i}\omega t}$. The geometry is divided into two domains, the canyon itself and a half space above it. The intersection between the two domains is denoted Γ. In the domains the Green functions are

$$G_1(\mathbf{r}|\,\mathbf{r_0},\omega) = \frac{c^2}{l_x l_y}\sum_n\sum_m \frac{\Psi_{n,m}(\mathbf{r})\Psi_{n,m}(\mathbf{r}_0)}{\Lambda_{n,m}(\omega_{n,m}^2(1+\mathrm{j}\eta)-\omega^2)} \tag{1}$$

$$G_2(\mathbf{r}|\,\mathbf{r_0},\omega) = \frac{-\mathrm{j}}{4}H_0^{(2)}(k|\mathbf{r}-\mathbf{r_0}|). \tag{2}$$

Equation (1) is valid in a 2D rectangular cavity with rigid walls, and (2) in a 2D free field. The room like Green function (1) is a summations of modes, where the resonance frequencies, mode shapes and modal weights are described by

$$\omega_{n,m} = \pi c\sqrt{\left(\frac{n}{l_x}\right)^2+\left(\frac{m}{l_y}\right)^2} \tag{3}$$

$$\Psi_{n,m}(\mathbf{r}) = \cos\left(\frac{n\pi r_x}{l_x}\right)\cos\left(\frac{m\pi r_y}{l_y}\right) \tag{4}$$

$$\Lambda_{n,m} = \int_\Gamma \Psi_{n,m}^2 dS. \tag{5}$$

The mode summation extends over all modes in theory, but in practice only a finite amount can be included. In this paper modes with resonance frequencies up to three times the frequency of interest are included in the summation. The damping is very important for the final result, and it is included as the factor η. The actual values used in this paper are described in section 2.3 below.

2.2 Matrix formulation

In order to calculate the sound pressure at any point the two domains must be coupled together. Here a brief description is included on how that is accomplished. For a more detailed description see [3].

The first step is to formulate the pressure in the two domains as a function of the pressure contribution from sources located in the respective domains together with a contribution from the boundary Γ, which is seen as a source distribution. The pressure can be calculated using

$$p = QG_1 + \int_\Gamma qG_1 dx \tag{6}$$

inside the canyon, where Q denotes the priamry source (e.g. due to the traffic), and

$$p = -2\int_\Gamma qG_2 dx \tag{7}$$

above it. For simplicity only one primary source is included in the formulas above, but the model can handle any number of (coherent) sources located in one or both domains. $q(x)$ denotes the source distribution over the boundary Γ. The factor 2 in (7) is due to the image source in the boundary which is considered to be rigid.

The next step is to divide the boundary into elements where q is constant over each element. Then the integrals in equation (6) and (7) can be split up into into a sum of integrals over each element. The width of the elements were set to $\lambda/10$. Finally the pressure at the center point of each element is expressed both using (6) and (7), and since they must be equal the following matrix equation is formed,

$$\mathbf{Aq} = \mathbf{b} \tag{8}$$

where

$$A_{i,j} = \int_{\Gamma_j} G_1(\mathbf{r}_j|\mathbf{r}_i)dx + 2\int_{\Gamma_j} G_2(\mathbf{r}_j|\mathbf{r}_i)dx \tag{9}$$

$$b_i = Q\int_{\Gamma_i} G_1(\mathbf{r}_0|\mathbf{r}_i)dx. \tag{10}$$

Solving this equation system for the unknown source strengths $\mathbf{q}$ is straight forward, but the integrals of the exterior Green function must be dealt with carefully. Here a numerical integration technique known as Gauss quadrature was used [4]. The integrals of the room like Green function can be performed analytically, so no numerical method is needed.

Finally the pressure at any position can be calculated using (6) or (7) depending on whether the receiver is inside the canyon or above it.

2.3 Damping in the canyon

The assumption of damping is essential for the sound levels inside the cavity. Assuming rigid walls and only including air absorption leads to surprisingly high values at interior resonances. A more realistic approach would be to include the reflection factors of typical façades and other materials normally present, but for the purpose of validating the model as such a simpler way is to use measured values from rooms.

In this paper the damping measured in a reverberation chamber at Chalmers is used. A logarithmic curve fitting of measured values gave

$$\eta(f) = 10^{-0.94} f^{-0.42} \tag{11}$$

within the frequency range 100 Hz – 1 kHz. However, preliminary measurements of damping in courtyards indicate that these values are too low.

Note that the use of the Green function (1) assumes that the damping is evenly distributed throughout the medium. The method can also be modified to include the effect of patches with nonzero absorption at the boundaries at the cost of an increased computational effort, see [3].

3 CALCULATION EXAMPLES

3.1 Single canyon

For the initial tests a canyon from Söder in Stockholm, Sweden, was choosen. The distance between the façades is about 11 m and the height of the nearby buildings around 18 m. One line source was positioned at the floor of the canyon ($y = 0$), slightly of center at $x = 5$ in order to exite modes with both odd and even n. The sound pressure level relative to free field was calculated using the above method in a grid of points, and the results are presented for two third octave bands in figure 2.

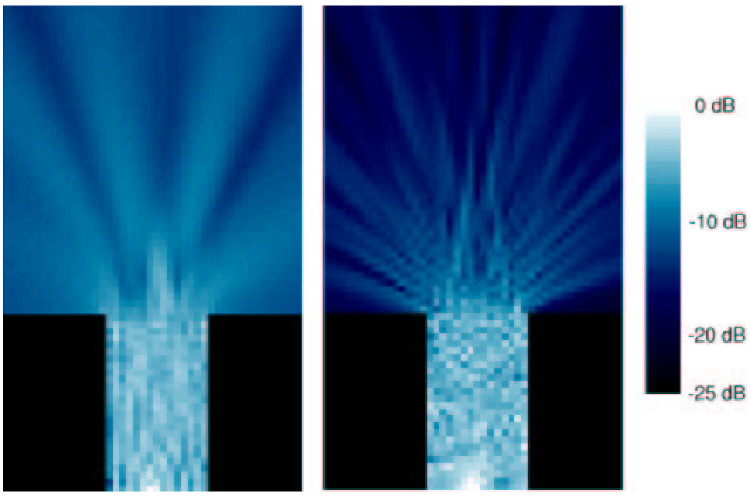

Figure 2: Color plot of the sound pressure level (uncal.) averaged over the third octave bands 125 Hz and 500 Hz.

3.2 Two canyons

The shielded courtyard was added to the model as a second canyon beyond the first. The width of the houses between the two canyons was set to 14 m, and the second canyon was 20 m wide, se figure 3. First the geometry without the second canyon was solved, and then the sources from the boundary were used as the primary sources for a second calculation, where only the second canyon was included. This approach will not take into account a posiible interaction between the canyons, which is assumed to be weak.

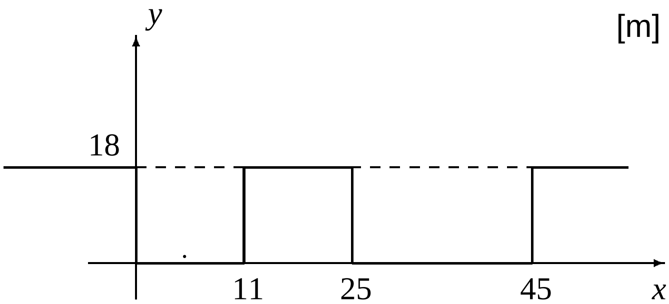

Figure 3: Sketch of the geometry for two canyons.

The calucations are displayed as color plots for the sound pressure level in figure 4 and 5 for the third octave bands 125 Hz and 500 Hz. The levels very close to the primary source have been truncated, since the level theoreticly goes to infinity at that point.

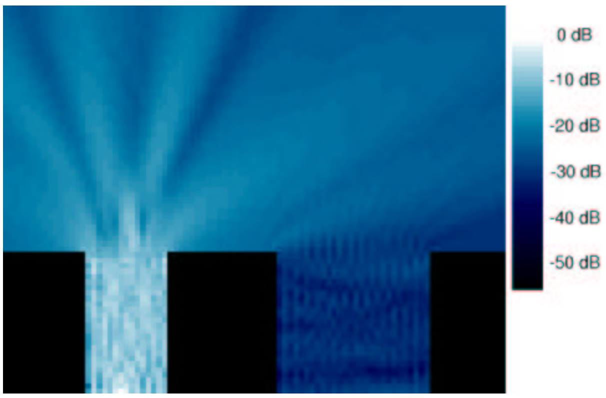

Figure 4: Results for two canyons, third octave band 125 Hz.

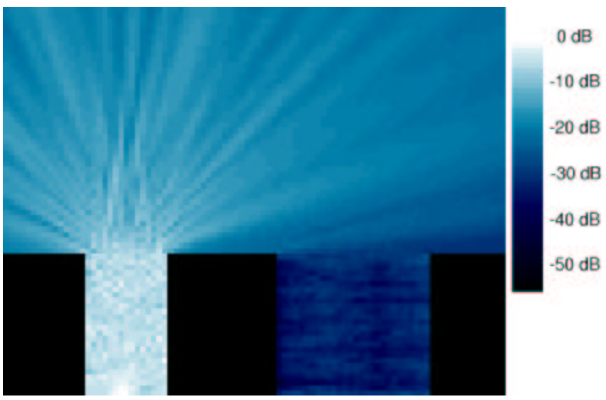

Figure 5: Results for two canyons, third octave band 500 Hz.

4 CONCLUSIONS

The level is relatively constant in both canyons, the variation with height and position within respective canyon is small. This is consistent with measurements from courtyards.

The level in the shielded canyon is not strongly dependent of the height of the shielding building. This is in contradiction to results obtained when a simplified diffraction theory is applied.

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