

# THE DYNAMIC BEHAVIOR OF THE ANTIVIBRATING SYSTEMS CONSISTING OF RUBBER ELEMENTS

PACS Reference: 43.40. Tm VIBRATION ISOLATORS, ATTENUATORS AND DAMPERS

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## ABSTRACT

This work points out the dynamic analyze of viscoelastic systems consisting of two types of rubber elements having specific elastic and viscous characteristics. Thus, the system main parameters having influence upon the vibration insulation degree are determined.

## INTRODUCTION

The insulation of vibrations and shocks resulted from road service, industrial activity or earthquakes represents a procedure of intended reduction of the forces transmitted to buildings, equipment and human body during these events.

The rubber antivibrating element application involves the elastic and dissipated modeling of the whole system having in regard that the compression rigidity for a single element has the form:

$$\tilde{K} = \lambda \tilde{G} = \lambda G_0 (1 + j\delta) \quad (1)$$

where

$G_0$  represents the shear modulus;

$\lambda = \left(1 + \beta \Phi^2\right) \frac{S}{h}$  the shape geometrical factor;

$\delta$  - the internal loss angle;

$j$  - the imaginary unit;

$\Phi$  - the shape factor defined as a ratio between the loaded area and the unloaded area;

$S$  - the cross section area;

$h$  - the active height of the rubber element;

$\beta$  - the multiplication factor depending on the rubber mixture nature.

The shear modulus  $G$  and the internal loss angle  $\delta$  have been determined in case of antivibrating elements consisting of four Romanian rubber receipts: AB4a, AB9, AB22 and AB31. Thus, the insulation characteristics in terms of force transmission have been analyzed in case of elastic one-stage system as well as in case of two-stage system.

## THE DYNAMIC BEHAVIOR OF ANTIVIBRATING SYSTEMS

Considering the case the elastic modulus and the internal loss angle does not depend on the excitation pulsation,  $G_\omega = G_0$  and  $\delta_\omega = \delta$ , in figure 1 is illustrated the vibration insulating system for one freedom degree model, having the transmissivity factor  $T_1$ .

Figure 2 presents the two-stage insulating model having the transmissivity factor  $T_2$ .

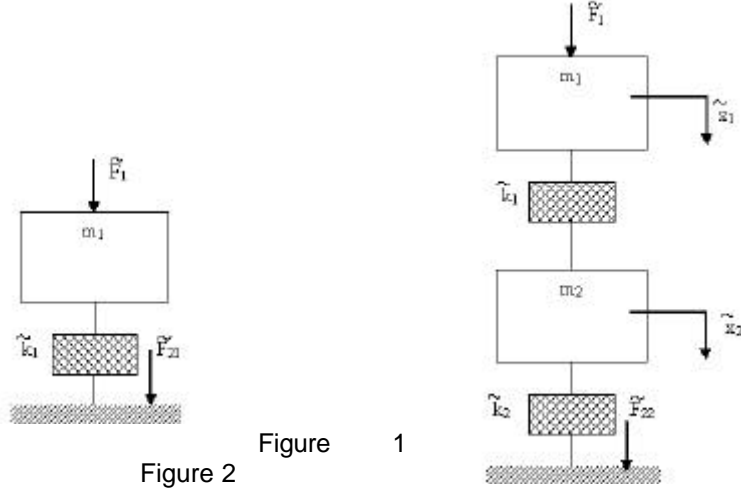


Figure 1

Figure 2

The transmissivity is determined by the following relation:

$$\tilde{T}_1 = \frac{\tilde{F}_{21}}{\tilde{F}_1} \text{ with } T_1 = |\tilde{T}_1| = \frac{F_{21}}{F_1} \quad (2)$$

in case of one-stage insulating system and

$$\tilde{T}_2 = \frac{\tilde{F}_{22}}{\tilde{F}_1} \text{ with } T_2 = |\tilde{T}_2| = \frac{F_{22}}{F_1} \quad (3)$$

in case of two-stage system.

The rubber elements are considered to be identically, meaning that the cross section area  $S_1 = S_2 = \dots = S_i = \dots = S_n$  and  $h_1 = h_2 = \dots = h_i = \dots = h_n$ . For these reasons the system global rigidity is:

$$K = \text{Re } \tilde{K} = \frac{G_1 G_2}{G_1 + G_2} (1 + \beta \Phi^2) \frac{S}{h} \quad (4)$$

where neglecting the mass  $m_2$  we obtain:

$$\omega_0^2 = \frac{G_1 G_2 S}{m_1 (G_1 + G_2) h} (1 + \beta \Phi^2) \quad (5)$$

The motion differential equations written in the complex form are:

$$\begin{cases} m_1 \ddot{\tilde{x}}_1 = \tilde{F}_1 - \lambda_1 \tilde{G}_1 (\tilde{x}_1 - \tilde{x}_2) \\ m_2 \ddot{\tilde{x}}_2 = \lambda_1 \tilde{G}_1 (\tilde{x}_1 - \tilde{x}_2) - \tilde{F}_{22} \end{cases} \quad (6)$$

with  $\tilde{F}_{22} = \lambda_2 \tilde{G}_2 \tilde{x}_2$ .

Basing on relations (6) the transmissivity factor  $\tilde{T}_2 = \frac{\tilde{F}_{22}}{\tilde{F}_1}$  is determined as:

$$\tilde{T}_2 = \frac{1}{\tilde{D}} \lambda_1 \lambda_2 \tilde{G}_1 \tilde{G}_2 \quad (7)$$

where

$$\tilde{D} = m_1 m_2 \omega^2 - m_1 (\lambda_1 \tilde{G}_1 + \lambda_2 \tilde{G}_2) \omega^2 - m_2 \lambda_1 \tilde{G}_1 \omega^2 + \lambda_1 \lambda_2 \tilde{G}_1 \tilde{G}_2 \quad (8)$$

Using the following notations  $\Omega = \frac{\omega}{\omega_0}$ ,  $\alpha = \frac{\lambda_2 G_2}{\lambda_1 G_1}$ ,  $\mu = \frac{m_2}{m_1}$  it results in:

$$T_1 = (1 + \delta^2)^{1/2} / \left[ (1 - \Omega^2)^2 + \delta^2 \right]^{1/2} \quad (9)$$

$$T_2 = (1 + \delta^2)^{1/2} / N^{1/2} \quad (10)$$

with

$$N = \left[ \mu \gamma \Omega^4 (2 + \mu)^{-1} - 2 \gamma \Omega^2 + 1 - \delta^2 \right]^2 + 4 \delta^2 (1 - \gamma \Omega^2)^2 \quad (11)$$

and

$$\gamma = (1 + \mu) / (2 + \mu)$$

$\gamma$  representing a parameter corresponding to the optimal ratio between the rigidities.

In case of negligible values for the internal loss angle  $\delta \cong 0$  and for  $\mu < 1$  the transmissivity factor values are very different in the excitation pulsation zone for  $\Omega > 5$  (figure 3). We have found the multistage antivibrating system significantly reduces the force transmitted to the foundation.

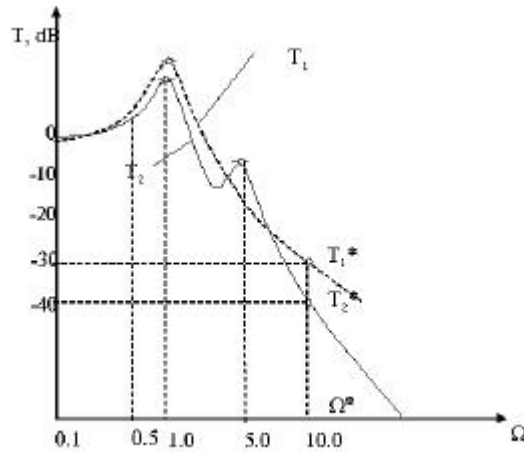


Figure 3

## CONCLUSIONS

The linear elastic model used for two-stage rubber antivibrating systems put into evidence the relevant diminution of the transmissivity.

On this basis one can design elastic systems having very low eigen frequencies (5...10 Hz), conducting to high vibration insulation degree in the range 90 ... 98%.

## REFERENCES

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