Tridimensionnal energetic coefficients as an investigation mean of refracted waves at the interface between anisotropic media

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ABSTRACT
During the ultrasonic nondestructive evaluation of materials, like multilayer carbon-epoxide composites, the behavior of the ultrasonic waves is determined by the anisotropy as well as the attenuation characteristics of the propagation media. Therefore the use of inhomogeneous waves is required and both real and imaginary parts of the data involved have to be considered. The developed numerical model allows the analysis of the reflection/transmission problem. The rotation of the incidence plane around the axis normal to the interface plane gives us tridimensionnal representations of the energetic coefficients leading to interesting information on the media.

INTRODUCTION

The use of ultrasonic non destructive evaluation is now widely spread in several fields of industry. As stated for a long time ago the behavior of the ultrasonic waves at the interface of samples requires to take into account the anisotropy as well as the attenuation characteristics of the propagation media in order to understand the signals obtained. Several computational models have been developed in our laboratory in that aim. The use of the representation of the reflection/transmission problem by mean of the complex slowness curves points out the evolution of the interaction of the waves at the interface.

THEORETICAL CONSIDERATIONS

Formalism

We developped since several years a propagation model involving inhomogeneous monochromatic plane waves. The displacement of such a wave can be written:

$$\vec{u}(\vec{X},t) = A\vec{P}\exp\left[i\omega\left(\vec{m}.\vec{X} - t\right)\right]$$

where A is the complex scalar amplitude, P is the complex polarization normalized according to the Hermitian product [1], $\vec{m}$ is the complex slowness vector noted $\vec{m} = \vec{m}' + i\vec{m}''$, where the real and imaginary parts correspond respectively to the propagating and attenuating parts of the wave. $\vec{X}$ is the position vector according to the followings coordinates: x_1x_2 is the plane of interface between the different media, x_1x_3 is the plane of incidence. ω is the pulsation or the angular frequency, t is time. Usually, x_3 represents the direction of highest degree of symmetry of the material before any tilt.

Method

According to developments detailed elsewhere [2], the motion equation leads to the Christoffel equation :

$$G_{ik} = C_{ijkl} m_j m_l - \rho \delta_{ik}$$

where G_{ik} stands for the elements of a symmetrical tensor of second-rank, called Christoffel tensor, C_{ijkl} is an element of the stiffness matrix and δ_{ik} is a component of Kronecker second-rank tensor. This system admits non-zero solutions only if :

$$\mathbf{det}(G_{ik}) = 0$$

which leads to a 6th order eigenvalue equation in m_1 and m_3, the components of the slownesses in the incidence plane. Once the incidence wave being chosen, the incidence angle yields to the value of m_3, and applying these considerations together with the boundary conditions at the interface, we obtain a system of equations in m_1. These equations are numerically solved and the determination of the six slowness vectors enables us to find the polarization vectors associated with these slowness vectors. It is well known that three solutions only are physically acceptable in each medium among the six obtained [3]. The selection of these waves is then performed according to an energetic criterion.

DISCUSSION

Energetic Considerations

The choice to perform in order to select the valid solution is a matter of discussion. The inhomogeneity characteristics of the fields cause the amplitude to rise infinity toward one direction, depending of the orientation of the inhomogeneity vector. In order to be coherent with the homogeneous case, it seemed to us that the better choice was the energetic criterion, i.e. the energetic flow vector of the transmitted waves has to point towards the transmission medium, no matter the incident angle is [4]. This can even be illustrated by the trivial case of an inhomogeneous wave reflecting upon an interface (Figure 1). According to the Snell-Descartes laws of refraction, the horizontal components of the slowness vectors are the same, and for a isotropic media the vertical components of the vectors are also equal. This means that one of the two fields has its inhomogeneous component, which is drawn in pink in the figure, such that the field increases when the observer moves away from the interface. Indeed, we must note that the Sommerfeld radiation criterion, which stands for the amplitude decay with distance from their cause, is not convenient for plane waves [5].

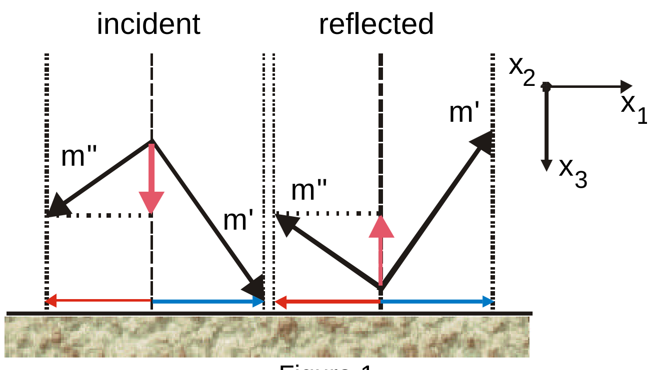

Figure 1

When the angle between the directions of inhomogeneity and propagation is negative, by respect to a direct triad (for example figure 2a, bottom), the radiation and energetic criteria lead to the same results. As a consequence, for positive values of this angle between m" and m' (figure 2b), it may seem to be paradoxical to have a transmitted field whose amplitude increases with the distance from the interface.

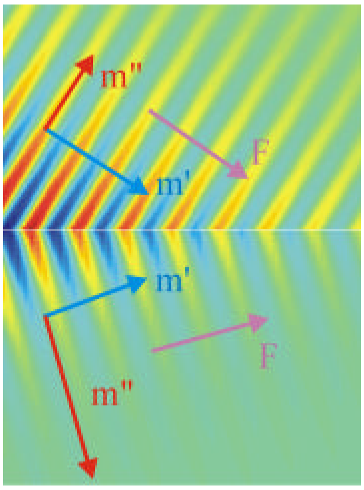

Figure 2 a

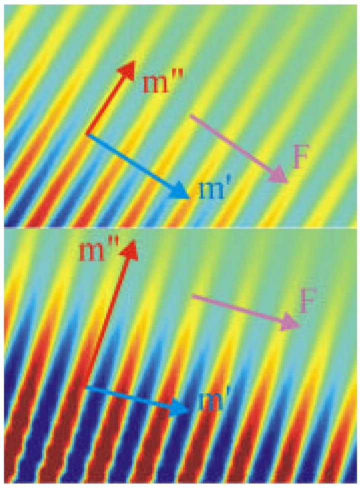

Figure 2 b

The point is that we have to consider that the existence of the field is not due solely to the incident/reflected field which is directly above the interface but to the whole field which exists in all the upper semi-space from an infinite time. Thus, it is necessary to take into account the totality of the phenomenon, in space and time [5].

Tridimensionnal Coefficients

Using our numerical model, we compute the different coefficients as well as slowness and energetic flux vectors for the reflection/transmission problem at the plane interface between two arbitrary media. The computation details being explained elsewhere [4,5], we only consider here the results obtained. Choosing the interface between water and nickel monocrystal as an example, the figure above (Figure 3) shows the geometry of the problem and the orientation of the planes involved by our study. For each relative orientation of the incident plane vs the crystallographic planes, we calculate the energetic coefficients at the interface and represent the reflection energetic coefficient. After a complete rotation of the incident plane, we obtain the tridimensionnal energetic coefficient as a function of the incidence angle, say θ, as well as the angular orientation around the vertical axis, say ψ. This result can be expressed by different means : cartographic, iso-levels, tridimensionnal surface plot.

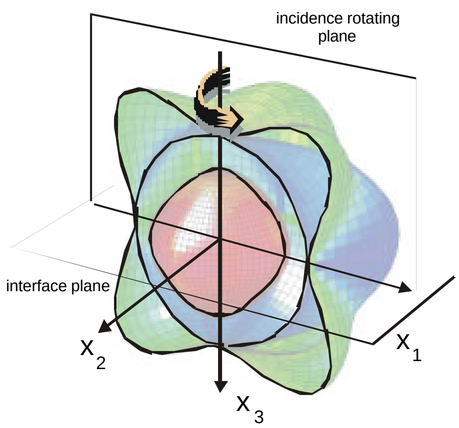

Figure 3

APPLICATION TO SEVERAL CASES

Water - Nickel Monocrystal

As a first example, we consider the case of the interface between water as the incidence medium and nickel monocrystal as transmission medium. Both water and nickel are assumed to be non absorbing. The figure 4 represents the intersection of the slowness surfaces with the incidence plane when $\psi = 0°$. The blue curves refer to the real parts of the slowness vector

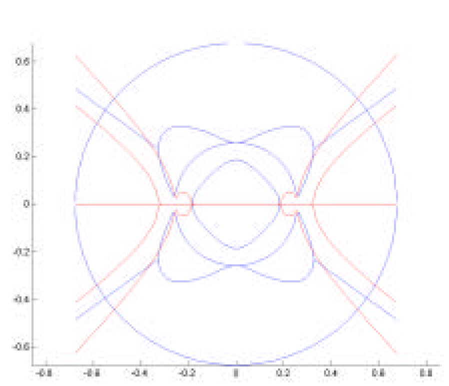

Figure 4

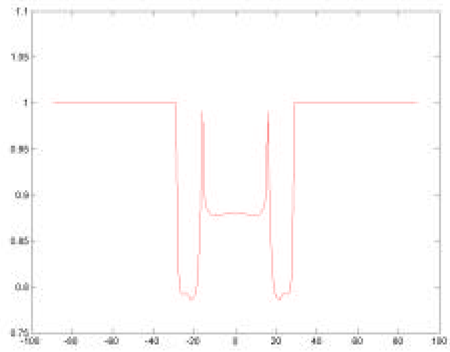

Figure 5

(circle : water), and the red curves are relative to the imaginary parts [6]. The figure 5 shows the reflection coefficient as a function of θ. Due to the symmetry of the media, this curve is obviously symmetrical by respect to the normal direction. The figure 6 presents the tridimensionnal energetic reflection coefficient. We can notice the wide extern red area which corresponds to the flat parts of the curve of the figure 5. It is interesting to note that the symmetry characteristics of the transmission media, namely 4-fold axis, is pointed out by the representation of this coefficient. The axis refer to the incidence angle in degrees and the direction $\psi = 0°$ corresponds to the horizontal axis. The figure 5 can be seen as an horizontal slice of the figure 6.

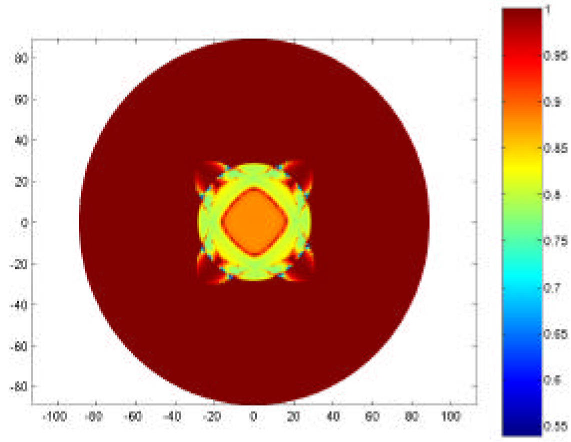

Figure 6

<u>Water - Carbon Epoxide</u>

We present some results in the case of the interface between water as incident medium and carbon-epoxide as transmission medium. We will consider the media non absorbing as well as absorbing. We only present the energetic reflection coefficients.

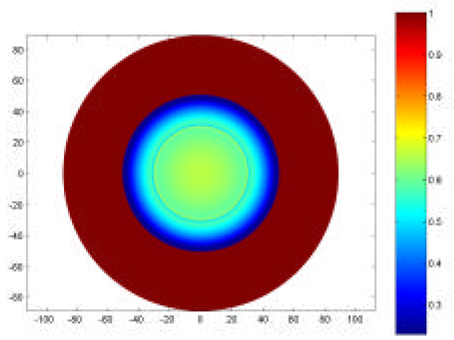

Figure 7

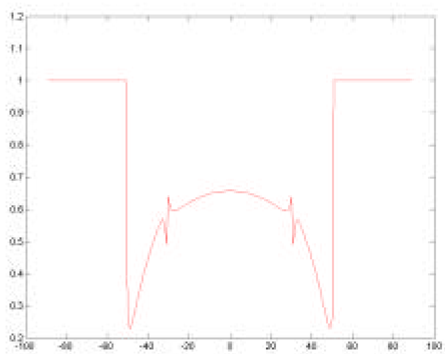

Figure 8

The figure 7 shows the case of non absorbing media. Due to the axi-symmetry of the media, we obviously obtain an axi-symmetrical figure. The figure 8 shows the bidimensionnal representation of the coefficient.

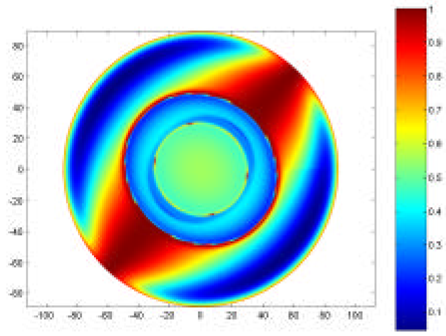

Figure 9

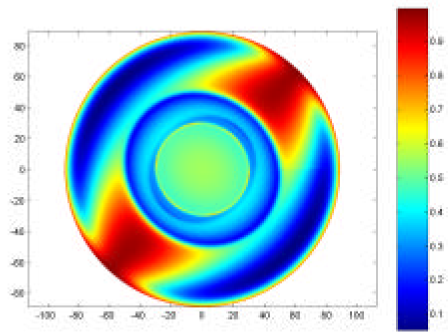

Figure 10

The figure 9 to 12 refers to the same transmission medium after a tilt of 15° around both the x_1 and x_2 axis. On figure 9 the carbon-epoxide is non absorbing and the incident wave is

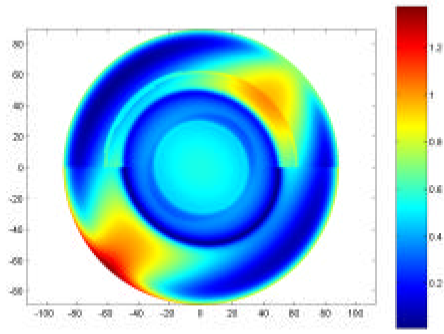

Figure 11

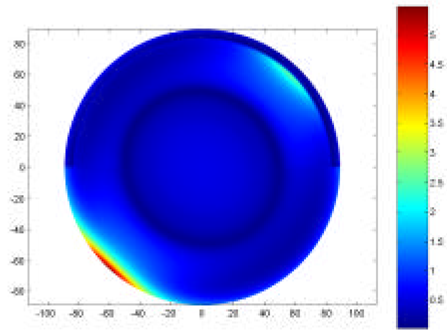

Figure 12

homogeneous. In the following figures the carbon-epoxide is absorbing and the incident wave is inhomogeneous for figure 11 and 12, with different evanescent component h, perpendicular to the propagation direction. For the figure 11 h=0.01µs/mm and for figure 12 h=0.05µs/mm. In

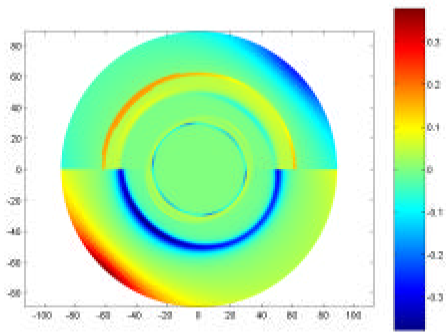

Figure 13

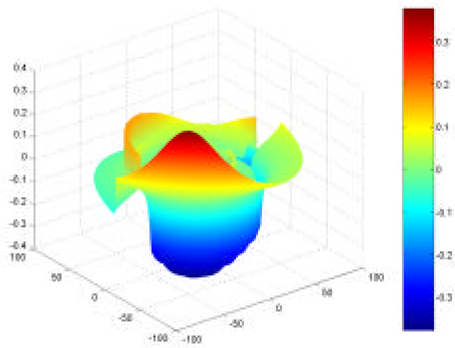

Figure 14

these last cases, the values of the coefficient can rise value above unity [7,8], as stated in the general inhomogeneous cases. In particular, for great values of inhomogeneity, like exposed by figure 12, the coefficients can reach important values for grazing incidences [9]. The figure 13 shows the difference between the coefficient in cases depicted by figures 11 and 10, i.e. the shift in the coefficient when the evanescent component changes from 0 to 0.01µs/mm. The figure 14 is a volumetric representation of the same quantity.

CONCLUSION

The use of a tridimensionnal angular representation of the energetic coefficient at the interface between media permits a global consideration of the phenomenon encountered. The differences obtained when the parameters are changed are easily pointed out by this kind of representation. The exploitation of this method is on the way in order to improve the modeling approach of the reflection/transmission problem at the interface between arbitrary anisotropic media.

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TABLE

Parameters used for the propagation media : volumetric mass ρ in kg/m^3, and viscoelasticity constants in GPa. The imaginary parts of the viscoelasticity constants are according to literature references.

	Water	Nickel	Carbon epoxide
ρ	1000	8620	1595
C_{11}	2.19-i0.00219	250	13.7-i0.130
C_{12}	2.19-i0.00219	155	7.1-i0.040
C_{13}	2.19-i0.00219	155	6.7-i0.040
C_{22}	2.19-i0.00219	250	13.7-i0.130
C_{23}	2.19-i0.00219	155	6.7-i0.040
C_{33}	2.19-i0.00219	250	126.0-i0.730
C_{44}	0	130	5.8-i0.100
C_{55}	0	130	5.8-i0.100
C_{66}	0	130	3.3-i0.045