THEORETICAL STUDY OF LAMB WAVE CONVERSION AT THE EDGE OF DIFFERENT ANGLES BEVELLED PLATES.

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ABSTRACT

We present a theoretical two-dimensional study of mode conversions that occur when an harmonic incident wave is reflected at the bevelled edge of a steel plate. A Lamb wave (A_1, A_0 or S_0 modes) is excited in a steel plate for different values of the frequency-thickness product. The energies of the reflected modes (A_1, A_0, S_0, S_1 or S_2 mode) are computed in order to know the reflected coefficients. Several results are presented for various bevel angles. These results are confirmed by a finite element computation.

INTRODUCTION

The Lamb waves are often used in the nondestructive structure testing because these waves propagate without attenuation. Then, since a few years, studies have been devoted to the reflection of Lamb wave normally incident at the free edge of a plate and to the incident mode conversion in a few reflected modes.

Some researchers have tried to explain this phenomenon. Firstly, Torvik[1] suggested that the waves which propagate in a plate should be the same as in an infinite plate. Afterwards, he adds the incident mode and the reflected modes to check the stress nullity at the end of the plate. Predoï [2] used this method to investigate the reflection of a Lamb wave at the free edge of a plate and at a weld. Gregory and al.[3] confirmed these results using a "projection method" which allows to compute the distribution of energy between reflected modes. Zhang and al.[4] studied this problem and checked the normal and tangential stresses nullity using the least squares principle. Rose [5] studied the mode conversion at the end of a plate by associating a boundary finite element computation with a superposition method of orthogonal modes. More recently, Morvan [6] used a Finite Element Method for the problem of Lamb mode conversion at the weld between two plates. These results are obtained with a straight cut plate.

In this paper, we numerically study the reflection of a Lamb mode at the end of a bevelled plate (angle from 70° to 85°), in the case of A_1, A_0 and S_0 incident modes. A finite element simulation entirely agrees about the numerical results.

THEORETICAL METHOD

The Lamb wave conversion study has been initiated by Torvik [1]. The method consists of the supposition that a Lamb wave propagates in a free loaded plate. This wave is reflected at the bevelled end of the plate and gives rise to few waves with both symmetries (Antisymmetric or Symmetric) which are observed versus the position x_1 [3].

In a plate, real, complex and purely imaginary modes can exist. We mean the modes whence the x_1 component wave vector $K_1=K'_1+j\,K''_1$ is real, complex or purely imaginary (*Figure 1*).

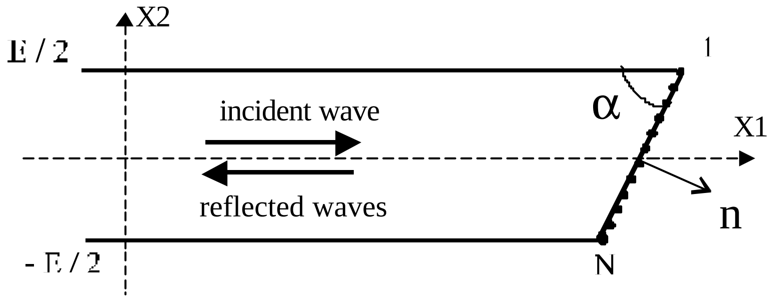
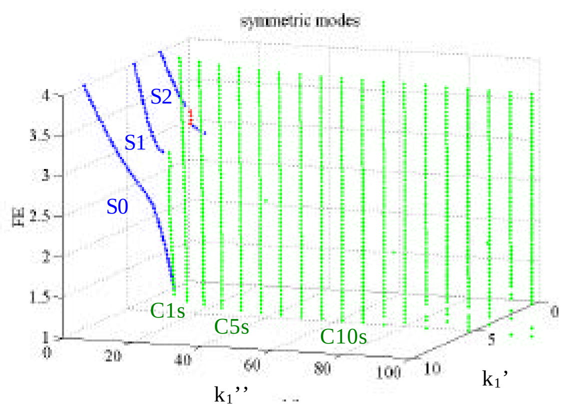


Figure 1 : Description of the plate.

For a given frequency-thickness product (FE), a finite number of real solutions have been found. These solutions are the Lamb waves. An infinite number of complex modes also exist. These modes have not physical meaning in the case of an infinite plate because their amplitudes become infinite when x_1 approaches $\pm\infty$, according to the sign of K_1'', the imaginary part of the wave vector. On the contrary, for a semi-infinite plate, exponentially damped complex modes can exist close to the extremity of the plate, decreasing from it.

We use a *Fortran* program to describe each mode. This program allows us the required complex wave vector K_1 at the wished frequency-thickness product and gives us the stresses of each mode when it propagates a power equal to 1W. The complex wave numbers are plotted in *Figure 2* and *3* as a function of FE (Symmetric and Antisymmetric waves).



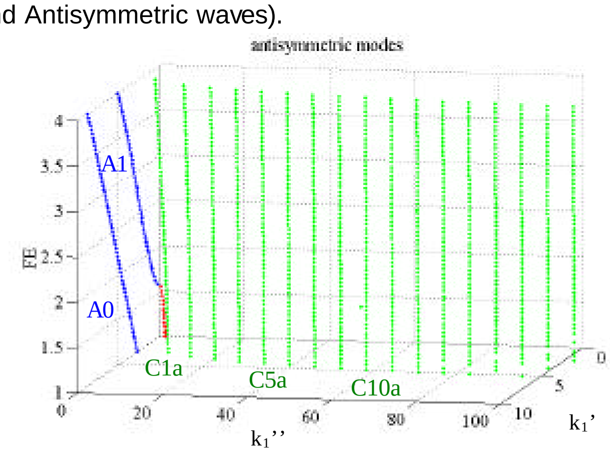


Figure 2: Dispersion curves of Symmetric waves. *Figure 3: Dispersion curves of Antisymmetric waves*.

At the bevelled edge, a sufficient large number of reflected modes must be used to express the boundary conditions at the end of the plate, *i.e.* the nullity of the components F_1 and F_2 of the Lamb wave force F on the inclined section. These components depend on the stresses T_{11} , T_{12} and T_{22}.

$$F_1 = T_{11}.n_1 + T_{12}.n_2 = T_{11}.\sin(\alpha) - T_{12}.\cos(\alpha) = 0$$

$$F_2 = T_{12}.n_1 + T_{22}.\ n_2 = T_{12}.\sin(\alpha) - T_{22}.\cos(\alpha) = 0$$

If we use the only real modes, the principle of energy conservation can not be checked. So we add to the first Lamb modes the complex modes whence the imaginary parts K_1'' of wave vector are the lowest.

To determine the wave amplitudes, we use a collocation method. We compute numerically, at N points, the incident wave stresses, *i.e.* $T_{11}^{inc}(i)$, $T_{12}^{inc}(i)$ and $T_{22}^{inc}(i)$ (for *i* from 1 to N). We suppose M reflected modes and we compute the stresses of these modes at N points $T_{11}^{m}(i)$, $T_{12}^{m}(i)$ and $T_{22}^{m}(i)$ (for *i* from 1 to N and m from 1 to M). The stresses nullity at the bevelled extremity is N times written:

$$\begin{aligned} F_1^{inc}(i)+\sum_{m=1}^{M} r_m.F_1^m(i)=0 \\ F_2^{inc}(i)+\sum_{m=1}^{M} r_m.F_2^m(i)=0 \end{aligned} \qquad i = 1 \text{ to N} .$$

The r_m coefficients are assigned to displacements and stresses of the m[th]-reflected wave and 1 to the incident wave. So, we dispose of a set of 2N equations (N equations for F_1 and N for F_2) which can be written as follow:

$$\begin{bmatrix} F_1^{inc}(1) \\ F_2^{inc}(1) \\ F_1^{inc}(2) \\ F_2^{inc}(2) \\ . \\ . \\ . \\ F_1^{inc}(N) \\ F_2^{inc}(N) \end{bmatrix} + \begin{bmatrix} F_1^1(1) & F_1^2(1) & . & . & . & F_1^M(1) \\ F_2^1(1) & F_2^2(1) & . & . & . & F_2^M(1) \\ F_1^1(2) & F_1^2(2) & . & . & . & F_1^M(2) \\ F_2^1(2) & F_2^2(2) & . & . & . & F_2^M(2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ F_1^1(N) & F_1^2(N) & . & . & . & F_1^M(N) \\ F_2^1(N) & F_2^2(N) & . & . & . & F_2^M(N) \end{bmatrix} . \begin{bmatrix} r_1 \\ r_2 \\ . \\ . \\ . \\ r_M \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

The problem can be condensed in the matricial relation $F + Ar = 0$, where sizes of *F*, *A*, and *r* are respectively (2N x 1), (2N x M) and (M x 1). The matrix *r* must be found knowing *F* and *A*. To solve this relation, we use the transposed matrix A^T of the matrix *A*. So *r*, which contains the M r_m coefficients, is given by:

$$r = -\left(A^T A\right)^{-1} A^T F \ .$$

As soon as the problem is solved, we must check that, in return, reflected waves propagate the energy of the incident wave. Only reflected real modes have to be taken in consideration in this sum because complex modes do not transport energy. Indeed, far from the end of plate, only Lamb waves exist. If P_m and P_{inc} are the Poynting vector flow of the m[th] reflected and the incident modes ($P_m = r_m.r_m^*$, $P_{inc} = 1$, where r_m^* is the complex conjugate value of r_m). The reflection coefficients can be written: $R_m = \dfrac{P_m}{P_{inc}} = r_m.r_m^*$. So the energy evaluation is: $P_{inc} = \sum_{m=1}^{M_L} P_m = \sum_{m=1}^{M_L} R_m = 1$, if M_L is the number of Lamb waves. To ensure this equality with an adequate precision, we must find a number N of complex waves in order to converge towards a single solution, whereas the number ML of Lamb modes is from 2 to 5 in the frequency range.

FINITE ELEMENTS METHOD (FEM)

In order to verify the theoretical results, computations are made with the ANSYS F.E.M. code. The studied stainless steel plate is 40 mm long and 2 mm thick and its characteristics are: $E=2{,}0043.10^{11}$ $N.m^2$ (Young Modulus), $\upsilon=0{,}29$ (Poisson coefficient), $\rho=7800$ $kg.m^{-3}$ (density). A two-dimensional mesh is modelled (Ox_1x_2) to describe the Lamb wave propagation. The limited plate is regularly discretized in rectangular elements with 400 nodes in length and 20 in depth. So, node density is more than twenty elements per wavelength.

To generate a Lamb mode in the plate with the F.E.M., a transient analysis method is performed. The normal and tangential components of the theoretical Lamb wave displacements are imposed at the end plate nodes (30 period bursts are applied). We collect the temporal evolution of the normal displacements U_2 at the surface of the plate in order to obtain a time-space image. Two successive Fourier transforms are computed, one temporal and the other spatial, to observe the Lamb modes in the dual space (K, FE) [7]. A section of this representation is performed for a given thickness-frequency product. So, Lamb modes amplitudes are known versus wave number K. Displacement amplitudes are connected to the corresponding powers by the relation, which exists between normal displacements on surface plate and Poynting vector flow through a straight section of the plate. Let consider an example: the Lamb wave A_1 is incident, at a given frequency-thickness product FE. This incident mode gives rise to N reflected Lamb waves. We obtain the R_n energy reflection coefficients (n=1,…, N) as the ratio of the n^{th} reflected wave power P_n to the incident wave power P_{inc}: $R_n = \frac{P_n}{P_{inc}}$. Then we can execute the energy balance in the bevelled plate.

RESULTS

Our method has been used with stainless steel plates (c_l=5850 m.s^{-1}, c_t=3150 m.s^{-1}). The used frequency range is limited to FE=4MHz.mm. In this domain, a limited number of Lamb waves exist in the plate: A_0, S_0, A_1, S_1 and S_2. The waves other than A_0 and S_0 require a minimal frequency to propagate (1.57 MHz.mm for A_1, 2.72 MHz.mm for S_1 and 3.15 MHz.mm for S_2,). The S_1 mode behaviour in this frequency range is particular: when FE lies between 2.72 and 2.92 MHz.mm, the mode has two phase velocities corresponding to positive and negative energy velocities. The S_1 mode with a negative energy velocity is noted S'_1. We note S_1 the wave, with a positive energy velocity, which exists above FE=2.72 MHz.mm. For a given FE, the resolution requires a systematic use of the possible reflected Lamb waves in addition to the complex modes, which are necessary to obtain the stresses nullity at the end of the plate [1]. The number of these waves is not imposed; also, we varied it and noted that the solutions converge when the total number of waves exceeds 20. The presented results have been obtained with 45 modes; they converge for angle values between 70° and 90°. For smaller angles, the precision is less.

The results concerning the A_0, S_0 and A_1 incident waves at the 80° bevelled extremity are gathered in the *Figures 4*, *5* and *6*. For each FE value, the reflected coefficients are individually computed, their sum of the reflected Lamb waves is equal to 1 with a variation of 0.1%.

- We note that the geometrical dissymmetry of the plate extremity always involves the existence of reflected modes, which have not the incident mode symmetry.
- The change of symmetry can be important (*Figure 4* : A_0 incident mode, FE about 4 MHz.mm) but this is not a general case.
- Sometimes, reciprocal phenomena are observed. For FE included between 1.5 and 2.5 MHz.mm, the A_0 and A_1 modes reflection is an example. We note that A_0 mode is converted almost completely into A_1, whereas A_1 is converted into A_0 with the same percentages.
- In the domain 2.72<FE<2.92 MHz.mm, the S'_1 mode appears in all conversions with significant values.
- The reflection coefficients, versus FE, have fast variations in the vicinity of each Lamb wave cutt-off. We can note an exception for the A_0 and A_1 mode reflection: the reflection coefficient of the incident mode has strong variations in vicinity of FE=2.3 MHz.mm, whereas this value is not a Lamb wave cutt-off.

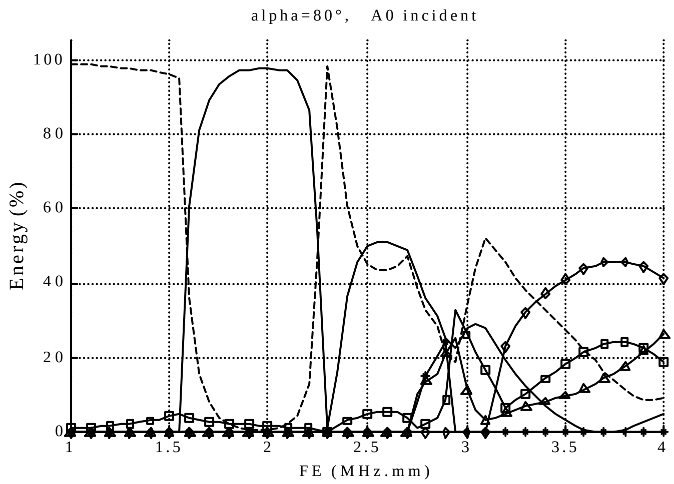


Figure 4: Energies of the different modes when A_0 is incident in the steel plate with α=80° (A_0 --, A_1 -, S_0 ð, S_1 ∆, S'_1 ∗, S_2 ◊).

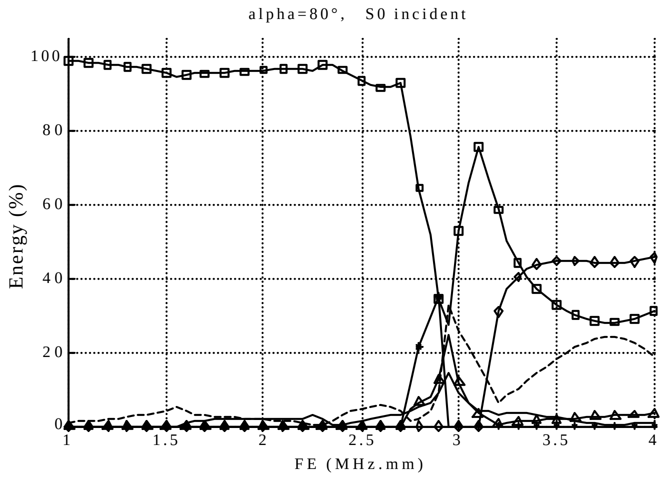


Figure 5: Energies of the different modes when S_0 is incident in the steel plate with α=80° (A_0 --, A_1 -, S_0 ð, S_1 ∆, S'_1 ∗, S_2 ◊).

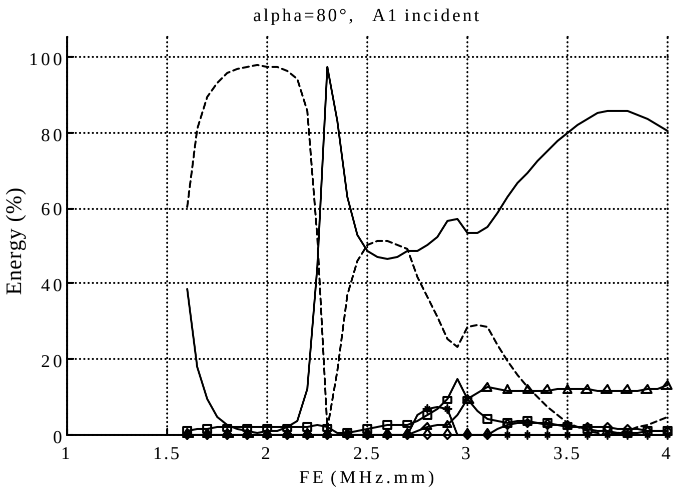


Figure 6: Energies of the different modes when A_1 is incident in the steel plate with α=80° (A_0 --, A_1 -, S_0 ð, S_1 ∆, S'_1 ∗, S_2 ◊).

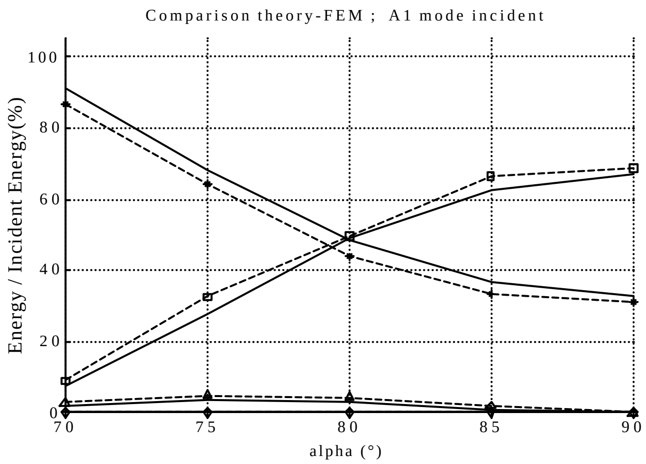


Figure 7: A_1 mode incident in the plate for different α at FE=2.7 MHz.mm,
*(A_0 ð, A_1 *, S_0 Δ, theory : solid lines , FEM : dashed lines).*

Some results have been compared to those obtained by the FEM. We note that our method is considerably faster than FEM and requires a smaller memory volume. So, the comparison between the two methods has only been realised for few cases (*Figure 7*). For FE=2.7 MHz.mm and with A_1 incident mode, we represent A_0, S_0, and A_1 reflected modes computed by both methods when α is between 70 and 90°. These results confirm the indications given by Rose [5] for α angles different from 90°.

CONCLUSION

We have shown that a Lamb wave is reflected at the bevelled end of a plate and produces Lamb waves observable far from the bevel. The reflected waves are a superposition of eigen modes of the plate checking energy conservation principle. This method, which gives the same results as the FEM, is perhaps not also universal but is easier and faster in the given problem. This method can be used in other Lamb waves conversions.

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