# ACTIVE CONTROL OF STRUCTURAL INTENSITY OF FLUID LOADED PLATES

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## ABSTRACT

In this paper, the active vibrational structural intensity (VSI) in an infinite elastic plate in contact with a heavy fluid is modeled by the Mindlin plate theory. This theory includes the shear deformation and rotatory inertia in addition to flexure. The plate is excited by a point force, which generates a vector active VSI field in the plate. The active VSI vector depends on the shear and moment, which are functions of the transverse displacement and the shear deformation angle. First, the Greens functions for the plate with and without fluid loading were developed. These were then used to develop expression for VSI. The displacement, shear deformation and VSI vector map are computed for frequencies below and above the coincidence frequency. Below the coincidence, a significant portion of source power is trapped in the plate in the form of VSI. Above the coincidence, a significant portion of the input mechanical power is leaked to the fluid in the form of radiated power, with a small portion propagating the farfield VSI.

## INTRODUCTION

Vibrational energy flow, in the form of vector structural intensity, defined as mechanical power flow per unit area, is one of the most useful methods of monitoring vibration and noise propagation. The structural intensity of a Mindlin plate model varies from that of a classical Euler-Bernoulli (E-B) plate model, and the difference is significant in the high frequency range where the Mindlin plate theory gives better predictions. The second part of this study is to control the structural intensity. This is to be achieved by locating the control forces at selected locations. The fluid-loaded plate excited by a point force is to be controlled by one or several point force actuators. The cost function to be minimized is the structural intensity at a reference point.

## FORMULATIONS

An infinite elastic plate with a point force excitation is modeled using the Mindlin plate theory in cylindrical coordinates. The plate is loaded with an infinite fluid on the upper side. Using the Mindlin theory, two simultaneous equations describe the motion of the plate in terms of the vertical displacement, w and the shear deformation factor, F.

$$(D\nabla^2 - G'h + \frac{\rho_s h^3}{12}\omega^2)\Phi - G'h\nabla^2 w = 0$$
$$G'h(\nabla^2 w + \Phi) + F_o\frac{\delta(r)}{2\pi r} - P_a = -\rho_s h\omega^2 w \qquad (1)$$

where “$^2$= Laplacian Operator, h=thickness of the plate, $r_s$=density of the plate,
$G'$ =adjusted shear modulus, $P_a$=acoustic pressure on the plate, D= bending stiffness,
$\Phi = \partial\psi/\partial r + \psi/r$ , and y=shear deformation.

The acoustic pressure, $p$, which acts on the plate, is described by the scalar wave equation.

$$\nabla^2 p + k^2 p = 0 \qquad (3)$$

where $k$=acoustic wavenumber=$w/c$ and $c$ = acoustic wavespeed

A continuity condition is needed to couple the normal acoustic particle velocity at the surface of the plate to plate velocity, $\dot{w}$ or

$$\left.\frac{\partial p}{\partial z}\right|_{z=0} = \rho_o\omega^2 w \qquad (4)$$

where $\rho_o$=acoustic fluid density.

In order to solve the Mindlin plate equations excited by a point force, the Hankel transform is utilized. The Hankel transform is applied to solve for the displacement and shear deformation. Solving the resulting algebraic equations and applying the inverse transform, the normalized displacement and the normalized shear deformation are derived below:

$$w_b = \frac{w}{h} = \int_0^\infty \frac{f_1(\rho)F_b}{f_2(\rho) - \frac{\varepsilon}{\sqrt{\rho^2-1}}f_1(\rho)}\rho J_0(rk\cdot\rho)d\rho = \frac{F_b}{2}\int_{-\infty}^{\infty}\frac{\rho\cdot f_1(\rho)H_0^{(1)}(rk\rho)}{f_2(\rho) - \frac{\varepsilon}{\sqrt{\rho^2-1}}f_1(\rho)}d\rho \qquad (5)$$

$$\psi_b = \frac{\psi}{kh} = \int_0^\infty \frac{\rho^3 F_b \frac{1}{rk}\int rkJ_0(rk\rho)drk}{f_2(\rho) - \frac{\varepsilon}{\sqrt{\rho^2-1}}f_1(\rho)}d\rho = \frac{F_b}{2}\int_{-\infty}^{\infty}\frac{\rho^2 H_1^{(1)}(rk\rho)}{f_2(\rho) - \frac{\varepsilon}{\sqrt{\rho^2-1}}f_1(\rho)}d\rho \qquad (6)$$

where $f_1(x) = 1 + K_s^2\Omega^2 x^2 - K_s^2 K_d^2\Omega^2$, $f_2(x) = (x^2 - K_s^2)(x^2 - K_d^2) - 1/\Omega^2$ ,
$\Omega = \omega/\omega_c$ , $K_s = \frac{c}{c_s} = \frac{c}{\sqrt{G\kappa^2/\rho_s}}$ , $K_d = \frac{c}{c_d} = \frac{c}{\sqrt{E/(1-\nu^2)\rho_s}}$ , $F_b = F_o/Dk^2$ ,
$\varepsilon = \rho_o/\rho_s\Omega^2 kh$ , $\omega_c = c^2\sqrt{\rho_s h/D}$ = classical coincidence frequency.

The time-averaged structural intensity after normalization for the Mindlin plate theory in cylindrical coordinates including the shear deformation y is derived below:

$$SI_b = \frac{SI}{Dk^3h^2\omega_c} = -\frac{1}{2}\mathrm{Re}[\kappa^2\frac{6(1-\upsilon)}{(kh)^2}(\frac{dw_b}{dkr} + \psi_b)\cdot i\Omega w_b^*] - \frac{1}{2}\mathrm{Re}[(\frac{d\psi_b}{dkr} + \nu\frac{\psi_b}{kr})\cdot i\Omega\psi_b^*] \qquad (8)$$

<u>Active Control of Structural Intensity</u>

In order to control the structural intensity, the object function of the control is chosen to be the structural intensity at (4p, 0) as a reference point. When the input source is located at the origin, three systems of controllers are used; one controller, two synchronized controllers and four synchronized controllers. Multiple controllers are located symmetrically with the origin. The control force has a real part and an imaginary part which can represent the magnitude and phase implicltly. Once the object function is developed, the steepest gradient algorithm is applied to find the minimum or optimal point. To guatantee positive definiteness of the object function, the SI is squared.

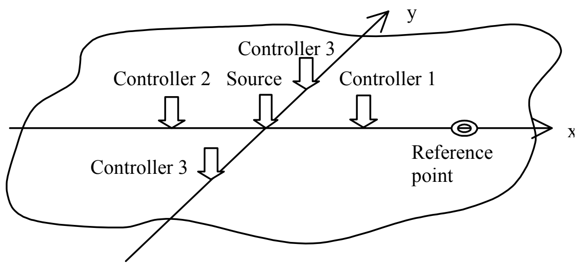
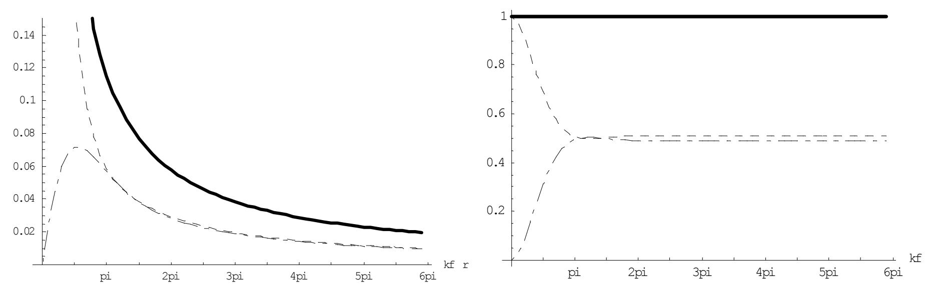



Figure 1. Geometry of plate with source and controllers

## RESULTS

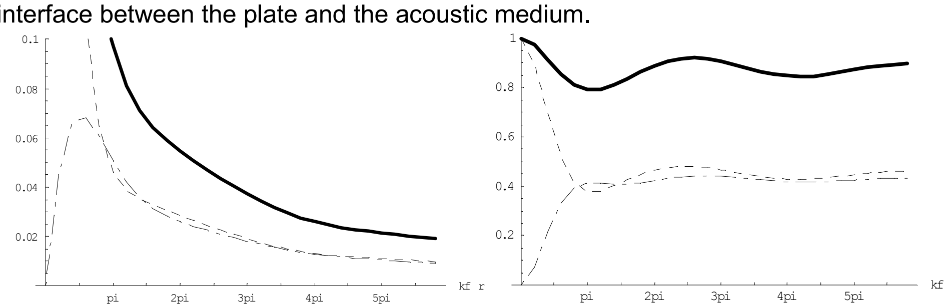
When the plate vibrates without an acoustic medium, the mechanical power is conserved in the redial direction unless damping exists. In Figure 2, the structural intensity decreases in the radial direction. When the structural intensity is multiplied by the circumference, the total power flow is conserved as shown. The SI plots in this paper are normalized by the mechanical source power. The horizontal axes of the plots are the distance normalized by the fluid-loaded structural wavelength, i.e. 2p means one full fluid-loaded structural wavelength.




Structural Intensity Total Power Flow

Figure 2. Unloaded Plate Structural Intensity (W=0.2)
(—: Magnitude, −−−: First Term. −−−−: Second Term)

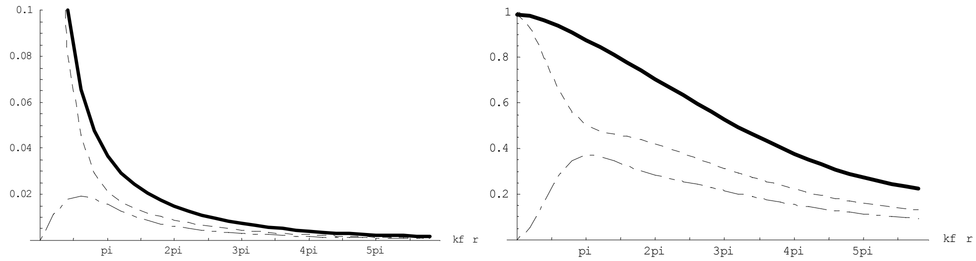
When the plate vibrates in contact with an acoustic medium below the coincidence frequency, a portion of the power injected into the plate radiates to the acoustic medium around the source, and the remainder is trapped in the plate. Unlike the unloaded plate, the SI integrated over the circumference displays the energy leakage around the source as shown in Figure 3. In the far-field, the outgoing power in the plate oscillates, which means that power exchange occurs on the interface between the plate and the acoustic medium.




Structural Intensity Total Power Flow

Figure 3. Loaded Plate Structural Intensity Below the Coincidence Frequency (W=0.2)
(—: Magnitude, −−−: First Term. −−−−: Second Term)

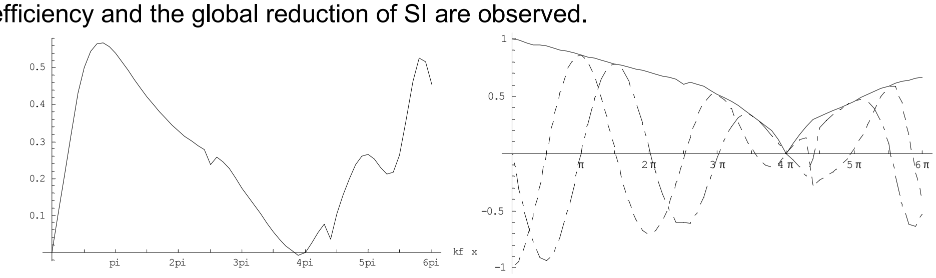
For the fluid-loaded plate above the coincidence frequency, any power injected into the plate transfesr to the acoustic medium while traveling to the far-field. Therefore, the structural intensity monotonically decreases in the far-field as shown in Figure 4.




Structural Intensity  Total Power Flow

Figure 4. Loaded Plate Structural Intensity Above the Coincidence Frequency (W=2)
(—: Magnitude, −−−: First Term. −-−-−: Second Term)

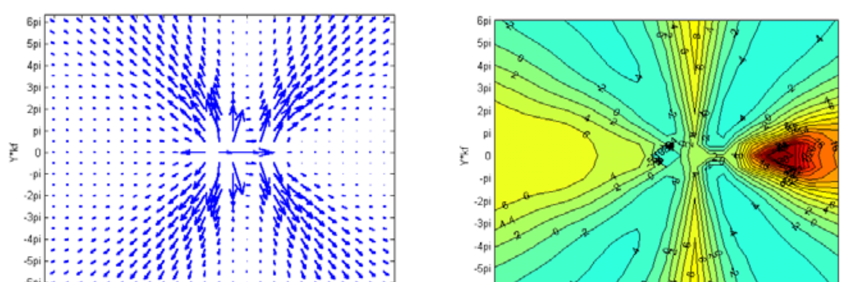
Figure 5 shows the control actuator power and the magnitude of the control force in order to control the SI below the coincidence frequency with the controller 1 in Figure 1. Because the SI at the reference point is controllable regardless of the number or the location of the controllers, the efficiency and the global reduction of SI are observed.




Control Power  Control Force

Figure 5. Control Power and Control force of One Controller when W=0.2
(—: Magnitude, −−−: Real Part, −-−-−: Imaginary Part)

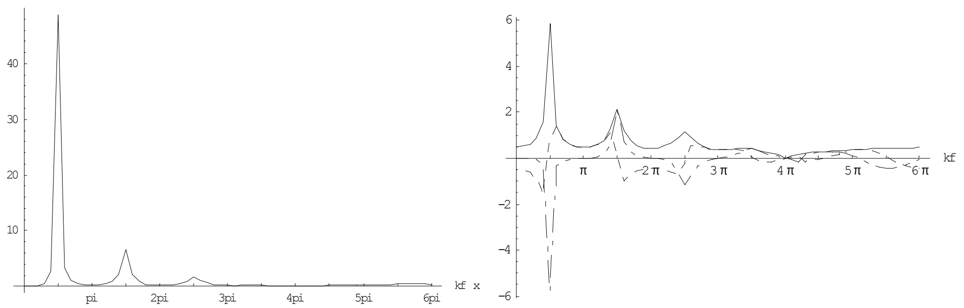
The control power is normalized by the uncontrolled input source power, and the power of one controller is lower than the uncontrolled input source power as shown above. The control force is also normalized by the magnitude of the source. When the controller is located at multiples of a half fluid-loaded structural wavelength, the source and the controller are out-of phase at the reference point as well as the input source location, and the SI decreases on ≤x-axis, ≤y-axis and their vicinities as shown in Figure 6. At other locations of the controller, the SI on the positive x-axis and small area around y-axis are reduced.




Structural Intensity  Reduction of SI

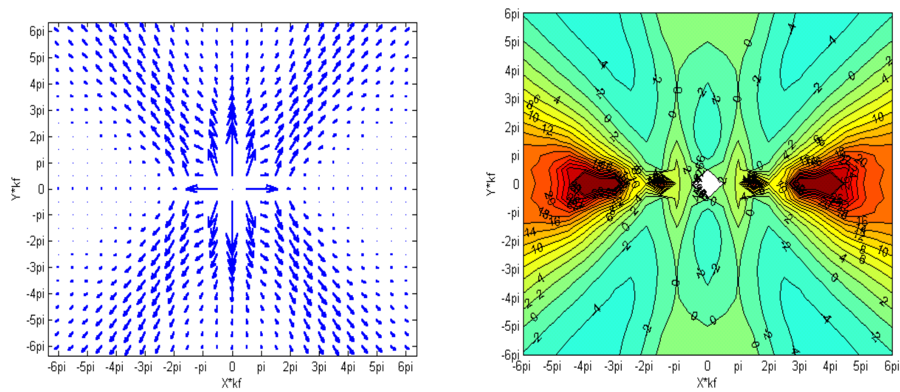
Figure 6. Structural Intensity and Reduction With One Controller at (2p, 0) when W=0.2

When two synchronous controllers are placed on x-axis symmetrically, such as the controllers 1 and 2 in Figure 1, the control power and the control force are shown in Figure 7. In this figure, an efficient control is feasible when the controllers are located at multiples of a half fluid-loaded structural wavelength or every $\pi$. Figure 8 shows the SI vector field map and the reduction of SI with the two controllers located at (≤2p, 0), one full fluid-loaded structural wavelength from the source. Because two controllers are synchronized, the results are symmetric to the y-axis. The SI is reduced on the x-axis and its vicinity, and small or no increment of SI on y-axis. In other areas, the SI is increased.




Control Power Control Force

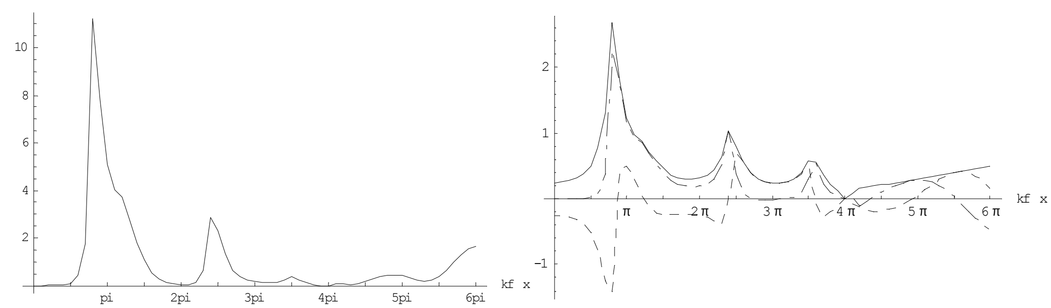
Figure 7. Control Power and Control force of Two Controllers when W=0.2
(—: Magnitude, – – –: Real Part, – - – - –: Imaginary Part)




Structural Intensity Reduction of SI

Figure 8. Structural Intensity and Reduction With Two Controllers at (≤2p, 0) when W=0.2

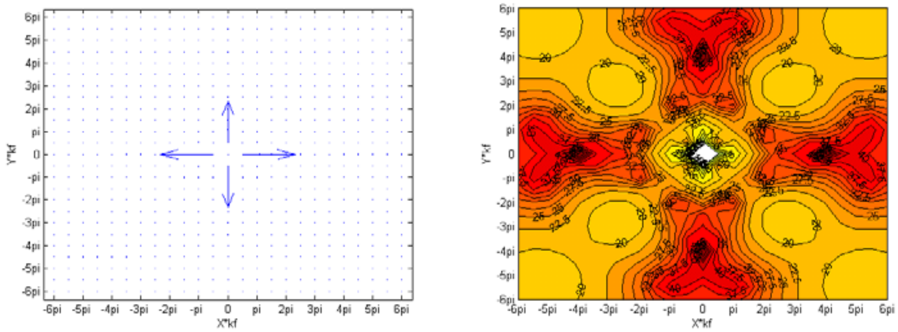
When the controllers are located other than multiples of half fluid-loaded structural wavelength, the SI reduction area decreases and the SI increases significantly. When four controllers are used such as controllers 1, 2 and 3 in Figure 1, the control power and the control force of each controller are displayed in Figure 9.




Control Power Control Force

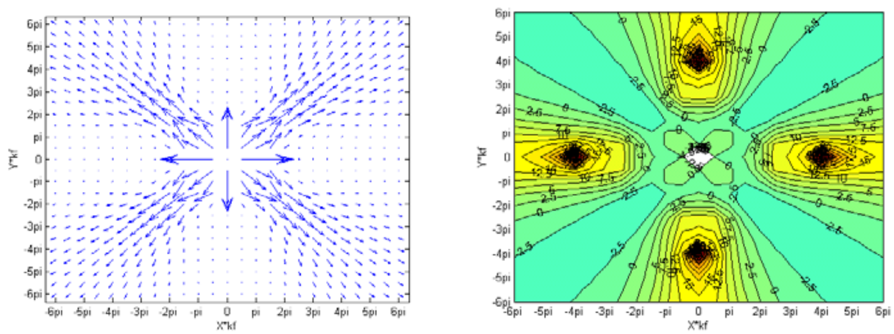
Figure 9. Control Power and Control force of Four Controllers when W=0.2
(—: Magnitude, – – –: Real Part, – - – - –: Imaginary Part)

When one or two controllers are applied for SI control at the reference point, the global control of SI is not feasible because the power from the source is rerouted to the area where the controllers are not placed. However, when four controllers located within a quarter fluid-loaded structural wavelength, the source impedance and type are altered so that the global control of SI is achieved. Figure 10 shows the results with controllers at (≤p/2, 0) and (0, ≤p/2). On other symmetric controller locations, the reductions occurs along the x and y-axes, yet no global reduction is achieved, see Figure 11. The results with an excitation frequency higher than the coincidence frequency are similar to the results below the coincidence frequency.



Structural Intensity Reduction of SI

Figure 10. Structural Intensity and Reduction With Four Controllers
at (≤p/2, 0) and (0, ≤p/2) when W=0.2



Structural Intensity Reduction of SI

Figure 11. Structural Intensity and Reduction With Four Controllers
at (≤2p, 0) and (0, ≤2p) when W=0.2

**CONCLUSIONS**

The structural intensity, the displacement and the shear deformation are derived and calculated using the Mindlin plate theory loaded by an acoustic medium. When the plate is unloaded by an acoustic medium, the power injected by the source is conserved in the radial direction. The plate coupled to an acoustic medium shows different behavior below and above the coincidence frequency. Below the coincidence frequency, a portion of the power injected by the source radiates into the acoustic medium around the source. Above the coincidence frequency, the structural intensity in the plate transfers to acoustic power in the far-field as well as in the near-field. Thus, the structural intensity decays in the far-field.

As far as the control at the reference point on the plate is concerned, the SI at a reference point is suppressible regardless of the location of controller/reference point or the excitation frequency. When global control is considered, the number and the locations of controllers are important factors. One controller is able to achieve a larger reduction area when the controller is located at multiples of a half fluid-loaded structural wavelength between the source and the reference point. Two synchronous controllers positioned symmetrically achieve a larger reduction area also, when the distances between the source and the controllers are multiples of a half fluid-loaded structural wavelength. Four synchronous controllers at symmetric locations result in significant global reduction when the controllers are located within a quarter fluid-loaded structural wavelength from the source. The controllers on other symmetric locations reduce the SI only on x and y-axes and their vicinities.