ANALYSIS OF A TIME-DOMAIN METHOD TO COMPUTE ULTRASONIC FIELD THROUGH INTERFACES

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ABSTRACT

This paper analyzes a time-domain method to compute the ultrasonic field radiated by broadband transducers when the acoustic field impinges interfaces. As first step, the impulse response velocity potential is determined at the interface using the Rayleigh integral. In a second step, the transmitted field is calculated considering that every elementary portion of the interface radiates a hemispherical wave. The aim of the work is to determine a compromise between the accuracy of the method and the computation time, having the temporal sampling of the transmitted signal and the spatial sampling of the aperture and the interface as variables.

INTRODUCTION

The investigation of the acoustic field produced by a broadband transducer through interfaces is an important tool for nondestructive evaluation by ultrasound, because in most of the cases liquid or solid wedges of diverse geometry exist between the transducer face and the structure.

Different methods have been developed to study the spatial-temporal characteristics of acoustic fields radiated by broadband transducers. One of these methods uses the spatial impulse response to determine the time-dependent pressure at a spatial point [1]. The starting point is the Rayleigh integral based on Huygens' principle from which, each point of a travelling wavefront can be considered as a secondary source of hemispherical disturbance. The acoustic field results from the superposition of hemispherical waves radiated by infinitesimal areas from the transducer. This method based on the Rayleigh integral is also used for calculation of pressure fields through interfaces [2] – [4].

In this paper a computational method is proposed, which is based on the spatial impulse response [1] and on the discrete representation computational concept [5]. Both the aperture transducer and the interface are considered as a finite number of elementary sources, each emitting a hemispherical wave.

In order to illustrate the application of the computational method to determining the acoustic field through interfaces, the radiation from a circular piston is initially considered in this work. Furthermore, a virtual interface in front of the circular piston is used to simulate the transmitted field and to compare it with an exact solution, considering the medium 1 equal to the medium 2. The exact solution of a circular piston mounted within an infinite baffle radiating into a medium is well-known [6] and will not be described here. The computational method is described for an arbitrary aperture in section.

THEORY

Consider an aperture with an arbitrary plane surface S_a embedded in an infinite rigid baffle that is in contact with a medium 1 (Figure 1). The acoustic pressure field radiated in an isotropic medium can be calculated in the time domain from the Rayleigh integral [1]:

$$p(\vec{r}_i,t) = \frac{r_1}{2p}\frac{\partial}{\partial t}\int_{S_a}\frac{v_n(\vec{r}_a,t-r_{ai}/c_1)}{r_{ai}}dS_a\,, \tag{1}$$

where $p(\vec{r}_i,t)$ is the incident acoustic pressure at the interface; r_1 and c_1 are, respectively, the density and propagation velocity of medium 1; r_{ai} is the distance from the elementary area dS_a located at $\vec{r}_a$ to the point located at $\vec{r}_i$; S_a indicates the surface of the radiating aperture; and $v_n(\vec{r}_a,t)$ is the normal velocity in each point of the aperture.

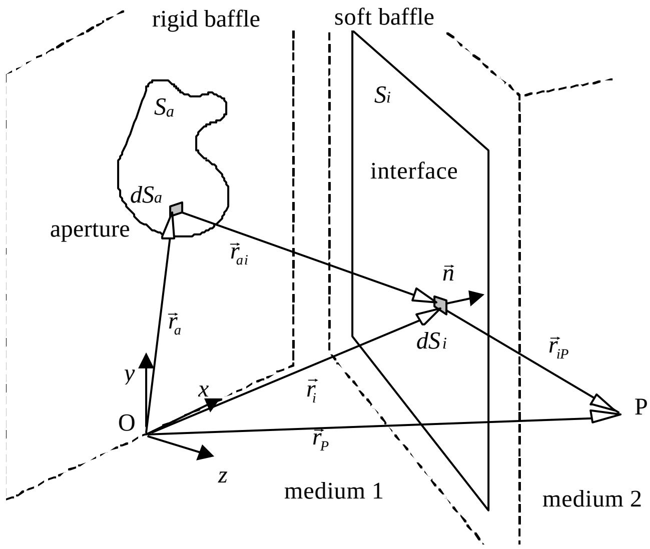

Figure 1. Geometry used to calculate the acoustic field through a plane interface.

The interface between media 1 and 2 is approximated to a finite interface, large enough to intercept the radiated field. Now considering that the acoustic pressure at this interface is known from equation (1), it is possible to calculate the refracted acoustic field applying the Rayleigh-Sommerfeld integral to the interface considering that it is embedded in an infinite soft baffle [7]:

$$p(\vec{r}_P,t) = \frac{1}{2p\,c_2}\int_{S_i}\frac{\cos(\vec{r}_{iP},\vec{n})}{r_{iP}}\frac{\partial}{\partial t}p(\vec{r}_i,t-\frac{r_{iP}}{c_2})dS_i\,, \tag{2}$$

where $p(\vec{r}_P,t)$ is the transmitted acoustic pressure through the interface; c_2 is the propagation velocity of medium 2; r_{iP} is the distance from the elementary area dS_i located at $\vec{r}_i$ to the field point located at $\vec{r}_P$; $\cos(\vec{r}_{iP},\vec{n})$ is the cosine of the angle between the normal vector $\vec{n}$ and the vector $\vec{r}_{iP}$; S_i indicates the surface of the finite interface; and $p(\vec{r}_i,t)$ is the defined pressure in each point of the interface.

The solution for $p(\vec{r}_P,t)$ presented in equation (2) can be simplified, assuming that the aperture is a uniform piston. Substituting equation (1) into (2), and after some calculations it follows that the transient field is determined by a temporal convolution between the excitation signal $v(t)$ and the velocity potential impulse response $h(\vec{r}_P,t)$:

$$p(\vec{r}_P,t) = r_1\frac{\partial v(t)}{\partial t} * h(\vec{r}_P,t)\,, \tag{3}$$

where the symbol * indicates the time convolution, and the velocity potential impulse response at the point P is defined by:

$$h(\vec{r}_P,t)=\frac{1}{c_2}\int_{S_i}\frac{\cos(\vec{r}_{iP},\vec{n})}{2\pi\, r_{iP}}\frac{\partial}{\partial t}h_a(\vec{r}_i,t-\frac{r_{iP}}{c_2})\,dS_i\,, \qquad (4)$$

where the velocity potential impulse response at the interface due to the aperture results in:

$$h_a(\vec{r}_i,t)=\int_{S_a}\frac{\delta(t-r_{ai}/c_1)}{2\pi\, r_{ai}}dS_a\,. \qquad (5)$$

In this work, computations are divided into two parts, using the approach proposed by Piwakowski valid for an arbitrary aperture [5]. In a first step, after dividing the aperture into a number of elementary areas, the velocity potential impulse response $h_a(\vec{r}_i,t)$ is calculated from equation (5) at the whole finite interface, which has also been approximated by a number of elements of small areas. In a second step, the velocity potential impulse response $h(\vec{r}_P,t)$ is calculated from equation (4) and the transmitted pressure field is obtained applying equation (3) at every field point P.

The time-averaged discrete impulse response [5] is the applied computational solution used to calculate the velocity potential impulse responses at the interface and at the point P, respectively, equation (5) and (4). The temporal sampling Δt is used constant during the calculation of the impulse responses, but the spatial samplings could be different between the aperture $\Delta x_a = \Delta y_a$ and the interface $\Delta x_i = \Delta y_i$.

RESULTS

In this section an implementation of the computational method will be presented for the case of a planar circular piston of radius 9.5mm radiating into a rectangular virtual interface of dimensions $L \times L$, as shown in Figure 2. An angular orientation α and an axial distance Z define the position of the interface relative to the referential $\mathrm{O}xz$ located at the center of the piston. All simulations were carried out taking both media 1 and 2 as water to permit a comparison with the exact solution of a piston radiating in water. An excitation signal of 1-MHz sine-wave single cycle was used (λ=1.5mm). The virtual interface was always located at the near field with Z=15mm and L=25mm.

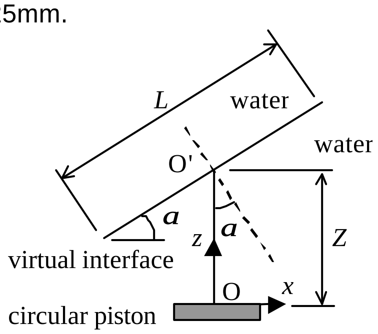

Figure 2. Illustration of circular piston radiating into a virtual interface.

Figure 3 compares the exact pressure responses with the respective model approach at different field points. Figure 3-a shows three different near-field points -$z = 20\mathrm{mm}$- at the acoustic axis and at 10 and 20 mm off axis. Figure 3-b shows the same results but at far-field conditions -$z = 60\mathrm{mm}$-. The model responses were calculated with $c\Delta t = 0.1\mathrm{mm}$, $\Delta x_a = \Delta x_i = 0.25\mathrm{mm}$, and $\alpha = 0°$. In all the cases the presented approach gives very good results except the P_3 case which, due to the geometry, is outside the "shadow" of the interface -Figure 4-.

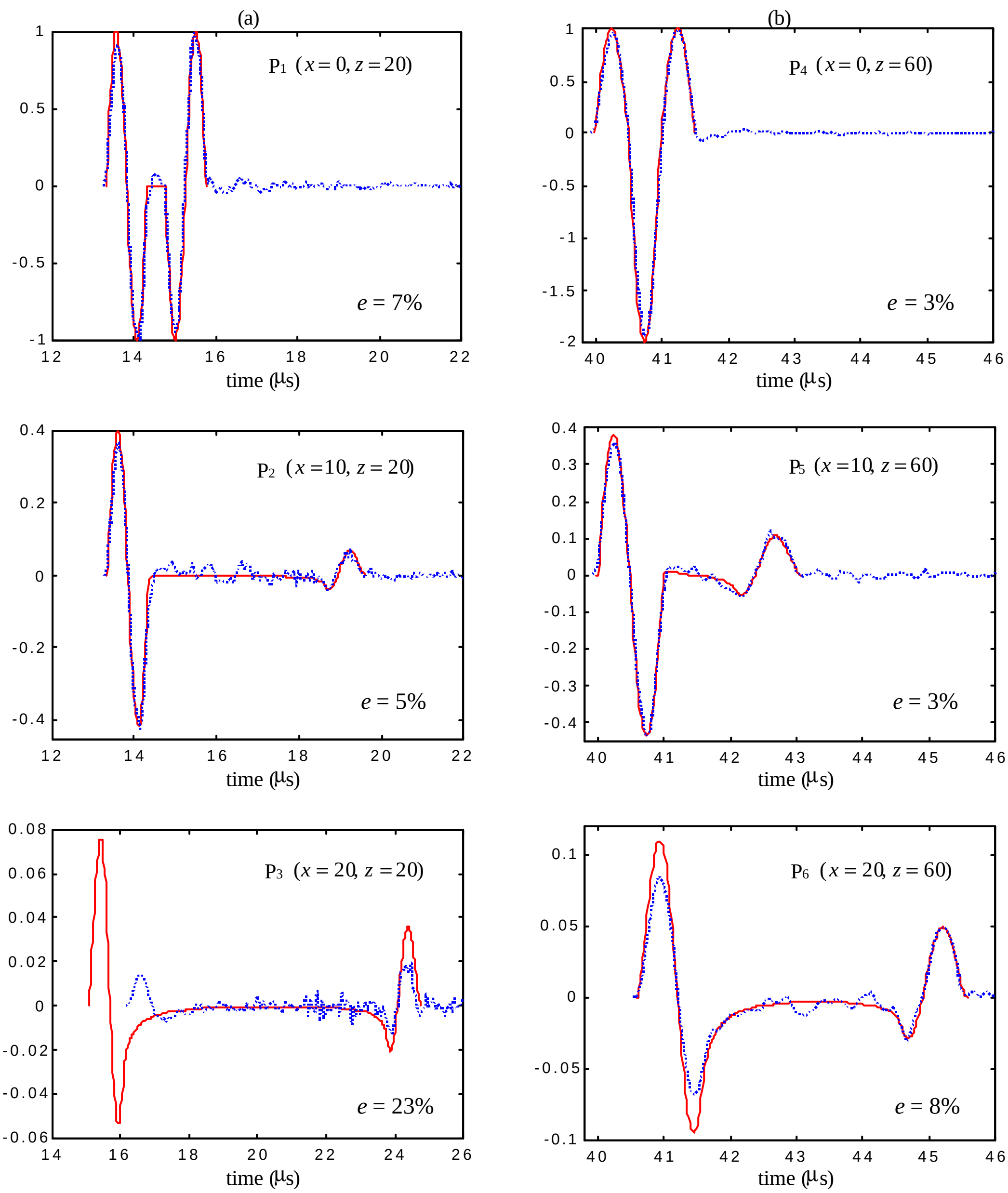

Figure 3. Comparison of exact (——) and approached (···) pressure responses to the regions: (a) near field in $z = 20\,\text{mm}$, and (b) far field in $z = 60\,\text{mm}$.

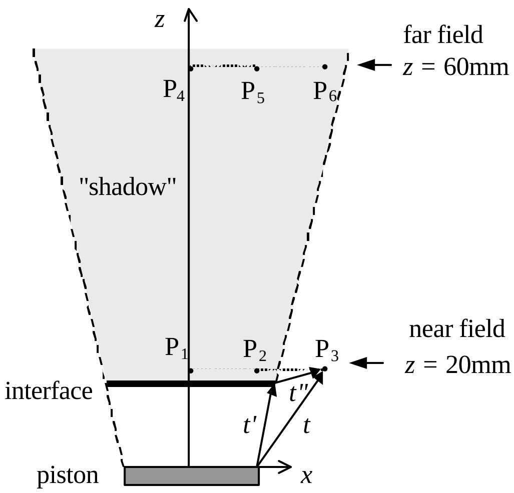

Figure 4.The "Shadow" of the interface and the six positions analyzed in Figure 3.

To give an indication of the approach accuracy, the error respect to the exact solution of the pressure responses has been calculated for every signal in the time domain:

$$e = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(s_E(i) - s_C(i)\right)^2}\ , \qquad (6)$$

where $s_E(i)$ is the signal computed by the exact solution, $s_C(i)$ is the signal computed by our method and N the number of samples. The error has been calculated for points at $z = 20\,\mathrm{mm}$ and $z = 60\,\mathrm{mm}$ which are in the range $|x| \leq 13\,\mathrm{mm}$. Figure 5 shows the maximum relative error found when varying the main computational parameters for three cases: (a) $\Delta x_a = \Delta x_i = 0.25\,\mathrm{mm}$, (b) $\Delta x_a = 0.25\,\mathrm{mm}$ and $c\,\Delta t = 0.1\,\mathrm{mm}$, and (c) $\Delta x_a = 0.5\,\mathrm{mm}$ and $c\,\Delta t = 0.1\,\mathrm{mm}$. As a general conclusion the error at near-field is bigger than at far-field conditions. Furthermore, it is noticed that an adequate temporal sampling improves the results. In relation with the spatial sampling, the precision of the computational method is limited by the number of elementary areas used to divide the aperture and the interface. To have an idea about the meaning of the error concept used, the errors between the approached pressure responses and the exact solution are displayed in Figure 3.

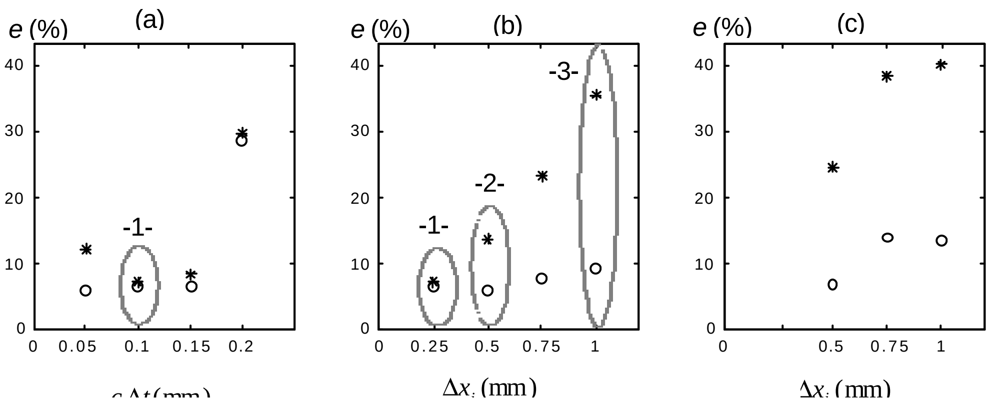

Figure 5. Maximum error e (%) of the pressure responses calculated at $z = 20\,\mathrm{mm}$ ($*$) and $z = 60\,\mathrm{mm}$ (o), for the range $|x| \leq 13\,\mathrm{mm}$ and the computational parameters: (a) $\Delta x_a = \Delta x_i = 0.25\,\mathrm{mm}$, (b) $\Delta x_a = 0.25\,\mathrm{mm}$ and $c\,\Delta t = 0.1\,\mathrm{mm}$, (c) $\Delta x_a = 0.5\,\mathrm{mm}$ and $c\,\Delta t = 0.1\,\mathrm{mm}$.

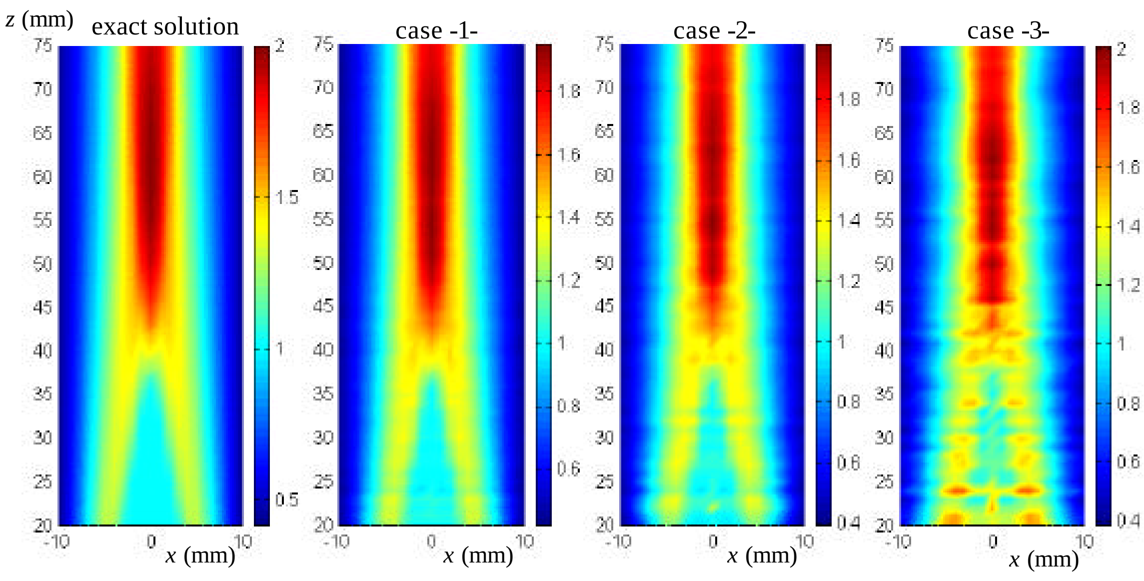

Figure 6. Simulated fields in he x-z plane. Comparison between the exact solution and three different computational cases (-1-, -2- and -3-) as described in Figure 5.

Finally, 2D simulated fields in the x-z plane have been calculated. Figure 6 is a comparison between the exact solution and three different computational cases. The cases are called -1-, -2- and -3- and the corresponding computational parameters are described in Figure 5 using ellipses. It can be observed that the main differences appear in the near-field region and when rough spatial sampling is used. Nevertheless big difference exists in computation time. For instance, using a 550 MHz Pentium III computer, case -1- takes 85 minutes whereas case -3- takes only 6 minutes.

CONCLUSIONS

Based on the impulse response and the discrete representation methods, a computational approach to calculate the acoustic pressure field through a plane interface by an arbitrary aperture has been developed. A closed-form analytic expression of the velocity potential impulse response has been obtained to describe the transmission through the interface. The number of elementary areas used to divide the aperture and the interface limits the precision of the computational method. The difference of the exact solution and its approximation can be minimized by means of an adequate choice of the temporal sampling. This method can be easily extended to arbitrary interfaces of complex geometry to predict the transmitted field. Furthermore, it permits the use of the transmission coefficients through interfaces.

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