BEHAVIOUR OF THE ELECTROMECHANICAL COUPLING FACTOR OF CYLINDER SHAPED PIEZOCERAMICS WITH DIFFERENT ASPECT RATIOS.

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ABSTRACT

The IEEE Standard on piezoelectricity defines the electromechanical coupling factor (k) in static conditions as the ratio between the energy converted from electric to mechanic form or vice versa. Recently, the authors have shown that this definition can be extended also to dynamic cases, by analysing a rod piezoelectric element by means of a 1D distributed model. In this work, the dynamic coupling factor is computed for cylinder shaped piezoceramic elements of different aspect ratios by using classical 1D models and a 3D analytical model. It is shown that it can be correlated to the effective electromechanical coupling factor (k_{eff}) for any aspect ratio.

INTRODUCTION

The electromechanical coupling factor *k* is a very useful parameter of piezoelectric materials because it synthetically characterizes the conversion from mechanical energy to electrical energy and vice versa. Several different definition of he *k* factor have been given in literature. The k_{ij}, also called the material coupling factor (k_{mat}) [1], refers to a fundamentally one dimensional geometry of the piezoceramic element and it is easily related to the energies involved in a dynamic transformation cycle. The effective coupling factor k_{eff} refers to an oscillating specimen of any geometry, but it is not clear in literature how it is related in a dynamic transformation cycle. Another definition of the coupling factor was given by Berlincourt *et al*. in their fundamental work on piezoceramics [2].This definition can be applied in static and dynamic situation and it can be computed, in principle, for any geometry of the piezoceramic element but, it does not a precise physical meaning and its relation with k_{eff}, which among all *k* factors is the only one that can be measured, is not clear.
In a recent work [3] the authors have shown that like in static situation also in dynamic situation it is possible to assign to the *k* factor the physical meaning of square root of ratio of energies. In this work, the dynamic coupling factor is computed for cylinder shaped piezoceramic elements of different aspect ratios by using a 3D approximated analytical model [4].

DEFINITIONS OF THE K FACTOR

The IEEE Standard on piezoelectricity [5] defines the static piezoelectric coupling factor as the square root of the ratio between the energy density converted from an electrical to a mechanical form w_c and the total energy density w involved in a transform cycle of a piezoelectric element:

$$k = \sqrt{\frac{w_c}{w}} \qquad (1)$$

The total energy density stored per unit volume by the element can be expressed by [6]:

$$w = \frac{1}{2} S_i T_i + \frac{1}{2} D_i E_i \qquad (2)$$

In the general case this expression is rather complicate; anyway if one dimensional geometries are concerned, the constitutive equations are simplified and this definition allows to give a physical meaning to the k factor.
Berlincourt defined the coupling factor as [2]:

$$k = \frac{U_m}{\sqrt{U_e U_d}} \qquad (3)$$

where U_e, U_d and U_m are the elastic, dielectric and mutual energy densities, respectively.
Even if it is possible to give a physical meaning to the quantity in (3), this energy density are not correlated to any energy conversion cycle; as a consequence the physical meaning of this k factor is different from that given in equation (1). However in the static case and for one dimensional geometries the Berlincourt definition gives the same result of the k factor defined by (1), and it can be applied also in the dynamic case, even if it becomes frequency dependent.
The effective electromechanical coupling factor as defined as [1]:

$$k_{eff} = \sqrt{\frac{f_p^2 - f_s^2}{f_p^2}} \qquad (4)$$

where f_s and f_p are the series and parallel frequencies of the equivalent lumped constants circuit which describes the piezoceramic specimen at each resonance frequency.
For low k_{eff} values the relation between k_{eff} and k_{mat} is :

$$k_{mat}^2 = \frac{p^2}{8} k_{eff}^2 \qquad (5)$$

while for high k_{eff} values this relation is plotted in [1,fig.9] for the main vibration modes.
The definition of the k factor as a square root of ratio of energies was extended to dynamic cases in [3]; for a lossless piezoelectric specimen insulated both electrically and mechanically the dynamic coupling factor k_W is defined as:

$$k_W = \sqrt{\frac{w_e}{w_k}} \qquad (6)$$

This definition is formally identical to equation (1) because the total energy in a cycle coincides with the kinetic energy w_k and the converted energy is the electric energy.
For distributed systems both w_e and w_p are frequency dependent and the dynamic coupling factor must be evaluated at the parallel resonance frequency:

$$k_W^2 = \lim_{w \to w_p} \frac{w_e}{w_k} \qquad (7)$$

1D MODELS

Figure 1 shows a piezoceramic rod (a) and a piezocermic disk (b), both poled in z-direction and with the flat surfaces electroded. These geometries can be used for describing three important one dimensional vibration modes: the ROD mode (fig.1a), the THICKNESS and the RADIAL mode(fig.1b). In the following the dynamic coupling factor k_W will be computed and compared with the k_{eff} for each of these modes.

Rod Mode

The constitutive equations for the rod mode are [2]:

$$T_{zz} = \frac{1}{s_{33}^D} S_{zz} - \frac{g_{33}}{s_{33}^D} D_z$$
$$E_{zz} = -\frac{g_{33}}{s_{33}^D} S_{zz} + \mathbf{b}_{33}^S D_z \tag{8}$$

and the wave equation is:

$$\mathbf{r}\frac{\partial^2 u_z}{\partial t^2} = \frac{1}{s_{33}^D}\frac{\partial^2 u_z}{\partial z^2} \tag{9}$$

By imposing stress-free boundary conditions and assuming a sinusoidal electrical excitation $D_z = D_0 e^{jwt}$, the solution of the wave equation is:

$$u_z = g_{33}\frac{\bar{v}}{\mathbf{w}}\left[\sin(kz) - \tan(\mathbf{q}/2)\cos(kz)\right]D_z \tag{10}$$

where $\bar{v} = \sqrt{1/\,\mathbf{r}s_{33}^D}$ is the wave propagation along the *z* direction, $k = \bar{v}/\mathbf{w}$ and $\mathbf{q} = kl$ .The expressions of the kinetic energy density w_k and of the electric energy density w_e are [3]:

$$w_k = \frac{1}{2}\frac{\mathbf{r}}{l}\int_0^l \left|v_z\right|^2 dz = \frac{1}{4}\mathbf{r}D_0^2 g_{33}^2 \frac{\bar{v}^2}{\mathbf{q}}\frac{\mathbf{q} - \sin\mathbf{q}}{\cos^2\frac{\mathbf{q}}{2}} \tag{11}$$

$$w_e = \frac{1}{2Al}C_0\left|V\right|^2 = \frac{1}{2}\frac{D_0^2}{\mathbf{b}_{33}^S}\left[\mathbf{b}_{33}^S - \frac{2g_{33}^2}{\mathbf{q}s_{33}^D}\tan(\mathbf{q}/2)\right]^2 \tag{12}$$

The limit of the ratio between these two energies when $\mathbf{w} \to \mathbf{w}_p$ (i.e. $\mathbf{q} \to \mathbf{p}$) after some manipulations is:

$$k_W^2 = \lim_{w \to w_p}\frac{w_e}{w_k} = \lim_{J \to p}\frac{w_e}{w_k} = \frac{8}{\mathbf{p}^2}\frac{g_{33}^2}{s_{33}^D \mathbf{e}_{33}^S} = \frac{8}{\mathbf{p}^2}\frac{g_{33}^2}{s_{33}^D \mathbf{b}_{33}^S} = k_U^2(\mathbf{w}_0) \tag{13}$$

Figure 2a shows the behaviour of w_k and w_e versus frequency for a PZT5A (Morgan-Matroc) specimen [7].

As it can be seen at the parallel frequency f_p both these energy densities tend to infinity. The ratio between these two energies, as well as the coupling factor k_U versus frequency is plotted in figure 2b: even if the behaviours of these functions are completely different, they assume the same value at the parallel resonance frequency $f_p = f_0$. This value slightly differs the k_{eff} value in agreement with the plot given by Berlincourt in [1,fig.9].

<u>Thickness Mode</u>

The constitutive equations for the thickness mode are [2]:

$$T_{zz} = c_{33}^D S_{zz} - h_{33} D_z$$
$$E_{zz} = -h_{33} S_{zz} + \mathbf{b}_{33}^S D_z \tag{14}$$

These equations are formally identical to eqs.8 and the wave equation is identical to eq.9. Therefore the results will differ from the previous ones only for some constants. We have:

$$w_k = \frac{1}{2}\frac{\mathbf{r}}{l}\int_0^l \left|v_z\right|^2 dz = \frac{1}{4}\mathbf{r}D_0^2\left(\frac{h_{33}}{c_{33}^D}\right)^2 \frac{\bar{v}^2}{\mathbf{q}}\frac{\mathbf{q} - \sin\mathbf{q}}{\cos^2\frac{\mathbf{q}}{2}} \tag{16}$$

$$w_e = \frac{1}{2Al}C_0\left|V\right|^2 = \frac{1}{2}\frac{D_0^2}{\mathbf{b}_{33}^S}\left[\mathbf{b}_{33}^S - \frac{2h_{33}^2}{\mathbf{q}c_{33}^D}\tan(\mathbf{q}/2)\right]^2 \tag{17}$$

The limit of the ratio between these two energies when $\mathbf{w} \to \mathbf{w}_p$ (i.e. $\mathbf{q} \to \mathbf{p}$) after some manipulations is:

$$k_W^2 = \lim_{w \to w_p} \frac{w_e}{w_k} = \lim_{J \to p} \frac{w_e}{w_k} = \frac{8}{p^2} \frac{h_{33}^2}{c_{33}^D b_{33}^S} = \frac{8}{p^2} \frac{e_{33}^2}{e_{33}^S c_{33}^D} = k_U^2(w_0) \quad (18)$$

Figure 3 shows the behaviour of w_k and w_e (a), and their ratio and k_U (b). The same considerations as those made for figure 1 hold.

Radial Mode

The constitutive equations for the radial mode are [5]:

$$\begin{aligned} T_{rr} &= c_{11}^p u_{r,r} + c_{12}^p u_r / r + e_{31}^p E_z \\ T_{qq} &= c_{12}^p u_{r,r} + c_{11}^p u_r / r + e_{31}^p E_z \\ D_z &= e_{31}^p (u_{r,r} + u_r / r) - e_{33}^p E_z \end{aligned} \quad (19)$$

By imposing stress-free boundary conditions and assuming a sinusoidal electrical excitation $D_z = D_0 e^{jwt}$, the solution of the wave equation is:

$$u_r = \frac{1}{jwD} \frac{J_1(kr)}{J_1(ka)} e^{jwt} \quad (20)$$

where $D = -\frac{pae_{33}^p}{e_{31}^p}\left[c_{11}^p\left(\frac{kaJ_0(ka) - J_1(ka)}{J_1(ka)}\right) + c_{12}^p + \frac{2\left(e_{31}^p\right)^2}{e_{33}^p}\right]$, $k = w/v^p$, $v^p = \sqrt{c_{11}^p / r}$.

The expression of the voltage V is:

$$V = \frac{B}{D} \quad (21)$$

where $B = j\frac{l}{wae_{31}^p}\left[c_{11}^p\left(\frac{kaJ_0(ka) - J_1(ka)}{J_1(ka)}\right) + c_{12}^p\right]$.

The kinetic and the electric energy densities are computed by means of the follow equations:

$$w_k = \frac{r}{a^2} \int_0^a |v_r|^2 r dr \quad (22)$$

$$w_e = \frac{e_{33}^p}{2l^2} |V|^2 \quad (23)$$

Due to the presence of the Bessel functions the energy densities w_k and w_e have no simple analytical expression; they are plotted versus frequency in figure 4a. Also for the radial mode both the energies tend to infinity at the parallel frequency f_p. Their ratio is plotted in figure 4b; the k_w value obtained is less than the k_{eff} value.

3D MODEL

In order to evaluate the dynamic coupling factor k_W for a cylinder shaped piezoceramic of any aspect ratio, an approximated 3_D model recently proposed by the authors [4] was used. This model was derived by assuming as solution of the wave equation system two orthogonal wave functions, i.e., the coordinate axes *r* and *z* are pure mode propagation directions ($u_r = u_r(r)$ and $u_z = u_z(z)$). As a consequence of this choice, the boundary conditions are not satisfied in every point of the external surfaces but only in an integral way. The comparison with results from one dimensional models have shown a very good agreement for rod and thickness mode, and an error of about 5% for the resonance frequency of the radial mode.

By imposing stress-free boundary conditions and assuming $D_z = D_0 e^{jwt}$, we have:

$$v_r = \frac{J_1(kr)}{J_1(ka)} v_1 \quad (24)$$

$$v_z = -\frac{v_3+v_2}{2}\frac{\sin(k_3 z)}{\sin(k_3 l/2)} + \frac{v_3-v_2}{2}\frac{\cos(k_3 z)}{\cos(k_3 l/2)} \tag{25}$$

$$V = \frac{2lh_{31}}{j\omega a}v_1 + \frac{h_{33}}{j\omega}v_2 + \frac{h_{33}}{j\omega}v_3 + \frac{I}{j\omega C_0} \tag{26}$$

Where the expressions of the v_1, v_2 and v_3 can be found in [4]. The kinetic and electric energy densities can be now computed as:

$$w_k = \pi\rho\int_0^l\int_0^a\left(|v_r|^2 + |v_z|^2\right)r\,dr\,dz \tag{27}$$

$$w_e = \frac{1}{2\pi a^2 l}C_0|V|^2 \tag{28}$$

The dynamic coupling factor k_W was computed for several aspect ratios *G=2a/l*. The results are shown in figure 5, where also the k_{eff} computed with the 3D model as well as with a FEM simulations, carried out with the code ANSYS 5.7, is shown. As can be seen the three curves present a very similar trend, even if for high values of G (radial mode) a greater difference between k_W and k_{eff} can be observed.

CONCLUSION

In this work the dynamic coupling factor, defined as the square root of the ratio between the electric and dynamic energy densities, has been computed and compared with the effective electromechanical coupling factor for the main one dimensional geometries of cylinder shaped piezoceramic elements, and a strong correlation between them was observed. The comparison has been then extended to cylinder shaped piezoceramics with different aspect ratios by using an approximated 3D model. Also in this case a correlation between the two coupling factors emerged, even if, due to the approximations at the basis of the 3D model, their values are much different than those obtained with 1D models for high diameter to thickness ratios.

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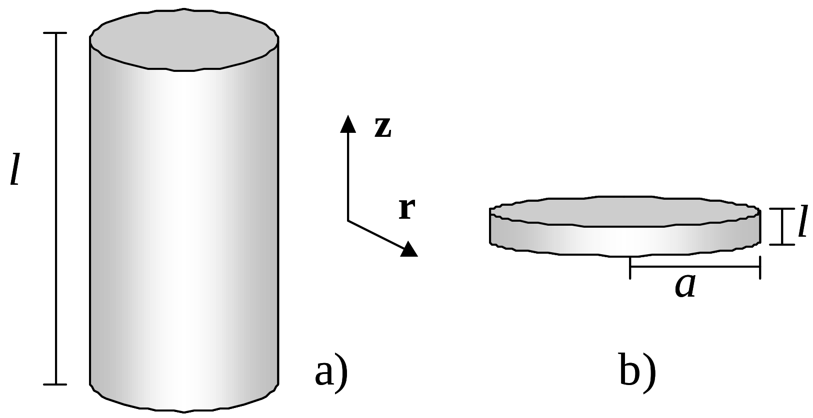

Figure 1: Piezpceramic cylinder shaped specimen: a) ROD, b) DISK.

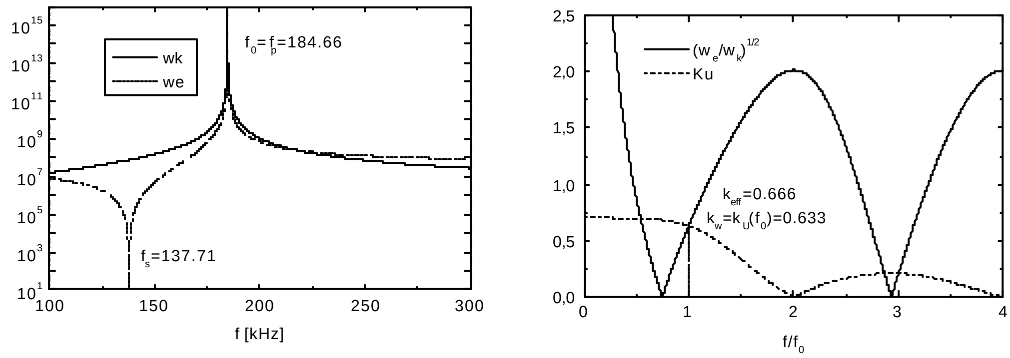

Figure 2: ROD mode; a)Energy densities w_k and w_e, b) w_e/ w_k and k_U

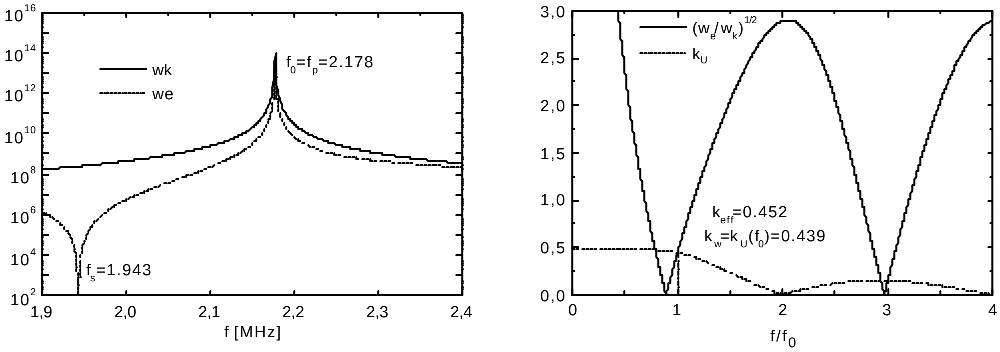

Figure 3: THICKNESS mode; a)Energy densities w_k and w_e, b) w_e/ w_k and k_U

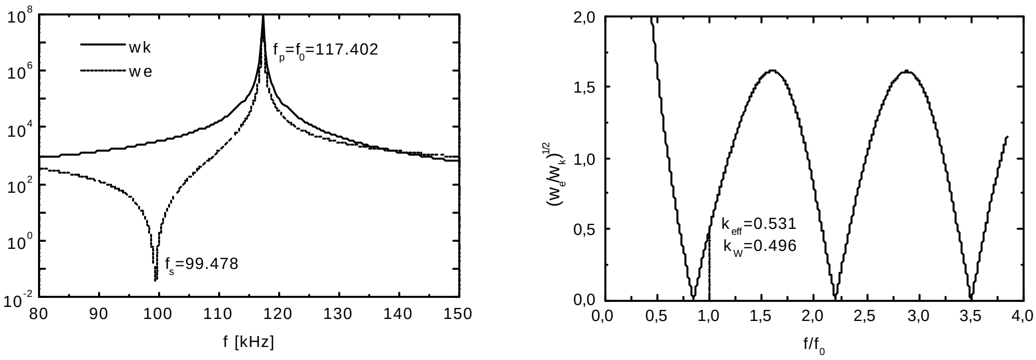

Figure 4: RADIAL mode; a)Energy densities w_k and w_e, b) w_e/ w_k

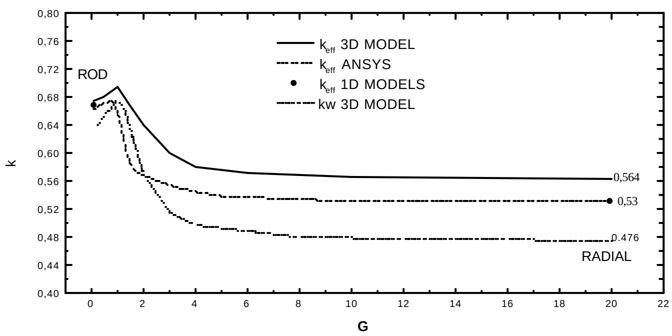

Figure 5: k_W and k_{eff} versus G