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NUMERICAL SIMULATION OF REACTIVE SILENCERS

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SUMMARY

This work shows the application of a finite difference algorithm to simulate the unsteady compressible flow in reactive silencers in order to obtain their sound attenuation. The numerical method solves the conservation equations of mass, momentum and energy. The method is applied to three different configurations of silencers including the expansion chamber, the Helmholtz resonator and the quarter-wave resonator. Comparison for the transmission loss between the theoretical and the calculated values are in good agreement for these geometries.

INTRODUCTION

Sound is present in daily life; it is a medium of communication, it can be useful in several circumstances but it can also be an irritating nuisance, in which case we refer to it as noise.

During the last years, an increase of the noise pollution has arisen and paaralleIy, the benefits of a quiet environment are strongly appreciated. There is a preference for quieter products whenever possible and this fact brings the advantage that the quiet product has over their noisy competitors.

In this work, the behaviour of reactive silencers is studied. A one-dimensional algorithm has been developed to simulate the flow of these silencers and to calculate their attenuation. The flow in the silencer is governed by the one-dimensional wave equation for the propagation of the pressure fluctuations, that is:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

It can be shown that the pressure fluctuations have an space and time dependence described by functions f and g such that

$$p(x,t) = f\left(t - \frac{x}{c}\right) \qquad p(x,t) = g\left(t + \frac{x}{c}\right)$$

are solutions of the one-dimensional wave equation. Function f describes a pressure fluctuation which is travelling in the positive x direction, and g describes a pressure fluctuation which travels in the negative x direction. A particular form of function which describes harmonic waves travelling in the positive x direction is given by the complex representation of harmonic motion described by

$$f\left(t - \frac{x}{c}\right) = \mathrm{Re}\left\{A\, e^{j\omega\left(t - \frac{x}{c}\right)}\right\} = \mathrm{Re}\left\{A\, e^{j(\omega t - kx)}\right\}$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency of the fluctuations having a period T. $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ is known as the wavenumber and A is a complex number. Thus, the pressure fluctuation in the positive x direction can be represented as $p(x,t) = |A|\cos(\omega t - kx + \phi_A)$. The pressure fluctuation in the negative x direction can be represented in a similar way.

In ducts, both kinds of waves can be happen, so the real pressure fluctuation is represented as: $p(x,t) = \mathrm{Re}\{p(x)e^{j\omega t}\} = \mathrm{Re}\{(A e^{-jkx} + B e^{jkx})e^{j\omega t}\}$, where A is the positive wave amplitude and B is the negative wave amplitude.

Basically there are two main groups of silencers: dissipative and reactive. Dissipative silencers attenuate sound energy incorporating an absorbent material that is usually fibrous or porous, while reactive silencers reduce the noise by means of a change in the transversal section of the duct, inducing a wave reflection. The latter ones are found in the exhaust pipe of internal combustion engines, but also in ventilation and air conditioning ducts. Basic geometries for reactive silencers are the expansion chamber, the Helmholtz and the quarter-wave resonator.

GOVERNING EQUATIONS AND SILENCERS DISCRETIZATION

The objective of this study is to simulate the one-dimensional compressible flow in a reactive silencer with a finite difference technique. The numerical method, coherent with the work of Chapman, Novak and Stein (1982), is applied to the conservation equations of mass, momentum and energy, assuming a perfect gas law. For a compressible unsteady flow, these equations can be written as follows:

Conservation of mass: $$\frac{\partial \rho}{\partial t} + \nabla(\rho U) = 0$$

Conservation of momentum: $$\frac{\partial}{\partial t}(\rho U) + \nabla(\rho U U) + \nabla p - \nabla \bar{\tau} = 0$$

Conservation of internal energy: $$\frac{\partial}{\partial t}(\rho e) + \nabla(\rho U e) + p\nabla U - \bar{\tau} \div \nabla U + \nabla q = 0$$

With the perfect gas law: $$p = (\gamma - 1)\rho e$$

These equations are discretized in ducts of variable cross section using an explicit finite difference algorithm, obtaining the following expressions:

Equation for the cell density: $$\frac{\rho^{n+1}_{j+\frac{1}{2}} - \rho^{n}_{j+\frac{1}{2}}}{\Delta t} + \left(\frac{M^{n+\frac{1}{2}}_{j+1} - M^{n+\frac{1}{2}}_{j}}{V_{j+\frac{1}{2}}} \right) = 0$$

where the superscript represents the temporal index of an specific variable.

Equation for the velocity on the cell face:

$$\frac{M^{n+1}_j U^{n+1}_j - M^n_j U^n_j}{\Delta t} + U^n_{j+\frac{1}{2}} M^{n+\frac{1}{2}}_{j+\frac{1}{2}} - U^n_{j-\frac{1}{2}} M^{n+\frac{1}{2}}_{j-\frac{1}{2}} + \left(p^{n+\frac{1}{2}}_{j+\frac{1}{2}} - p^{n+\frac{1}{2}}_{j-\frac{1}{2}} \right) a_j + \tau^n_{w,j} a_{w,j} = 0$$

where M^n_j is the cell mass, $U^n_{j+1/2}$ is the cell velocity, $M^{n+1/2}_{j+1/2}$ is the cell mass flow, $p^{n+1/2}_{j+1/2}$ is the cell pressure, $\tau^n_{w,j}$ is the shear stress and $a_{w,j}$ is the cell area.

Equation for the internal energy of the cell:

$$\frac{M^{n+1}_{j+\frac{1}{2}} e^{n+1}_{j+\frac{1}{2}} - M^{n}_{j+\frac{1}{2}} e^{n}_{j+\frac{1}{2}}}{\Delta t} + \left(M^{n+\frac{1}{2}}_{j+1} e^{n+\frac{1}{2}}_{j+1} - M^{n+\frac{1}{2}}_{j} e^{n+\frac{1}{2}}_{j} \right) + p^{n+\frac{1}{2}}_{j+\frac{1}{2}} \left(U^{n+\frac{1}{2}}_{j+1} a_{j+1} - U^{n+\frac{1}{2}}_{j} a_j \right) -$$

$$- \tau^n_{w,j+\frac{1}{2}} a_{w,j+\frac{1}{2}} U^{n-\frac{1}{2}}_{j+\frac{1}{2}} + h_{T,j+\frac{1}{2}} a_{w,j+\frac{1}{2}} \left(T^n_{j+\frac{1}{2}} - T_\omega \right) = 0$$

where h_T is the total heat transfer coefficient.

The cell pressure is calculated using the equation of state: $$p^{n+1}_{j+\frac{1}{2}} = (\gamma - 1)\, \rho^{n+1}_{j+\frac{1}{2}} e^{n+1}_{j+\frac{1}{2}}$$

These equations are applied to all the cells in order to obtain the variables for the time step n+1; then variables at the time step n are replaced by the variables at n+1 and the process is repeated until convergence is reached.

The staggered mesh used divides a duct into cells of the same length. While vector quantities are located at the cell faces, (index j), scalar magnitudes are placed at the cell centre (index j±1/2) (Figure 1).

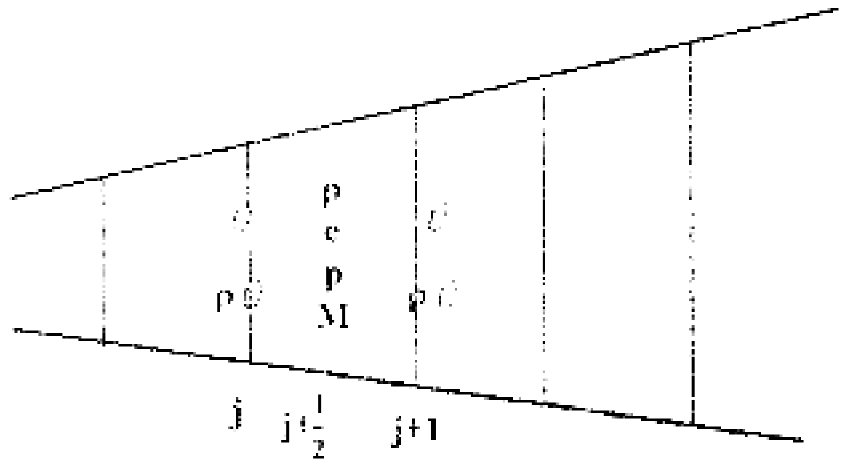

Figure 1

The noise source is modelled as an oscillating piston with a velocity given by $U_p^n = U_0 \cos(\omega t)$ where U_0 is the maximum velocity.

As the algorithm is explicit, the time increment is calculated with the Courant condition: $\Delta t < \frac{\Delta x}{c + |U|}$ where c is the velocity of sound, Δx is the cell length and U is the velocity of the cell faces. This increment of time is reduced by a safety factor to $\Delta t = 0.5 \frac{\Delta x}{c + |U|}$.

The expansion chamber is modelled as a duct and divided in cells of constant length Δx and variable cross section. At both ends the cross-section is smaller compared with the silencer cross section (Figure 2).

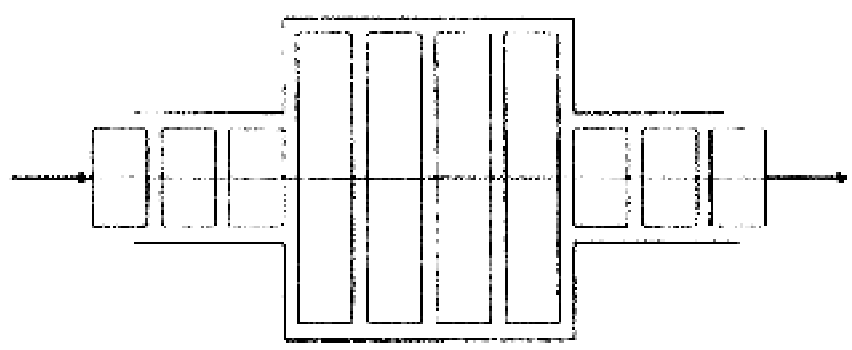

Figure 2

The Helmholtz (Figure 3) and the quarter-wave (Figure 4) resonators are modelled as two ducts connected in a junction and divided in cells of equal length Δx and variable cross section.

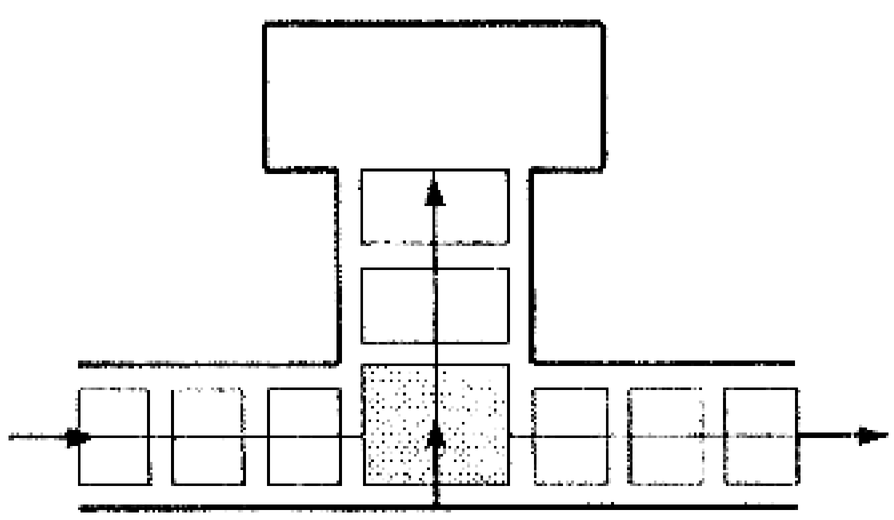

Figure 3

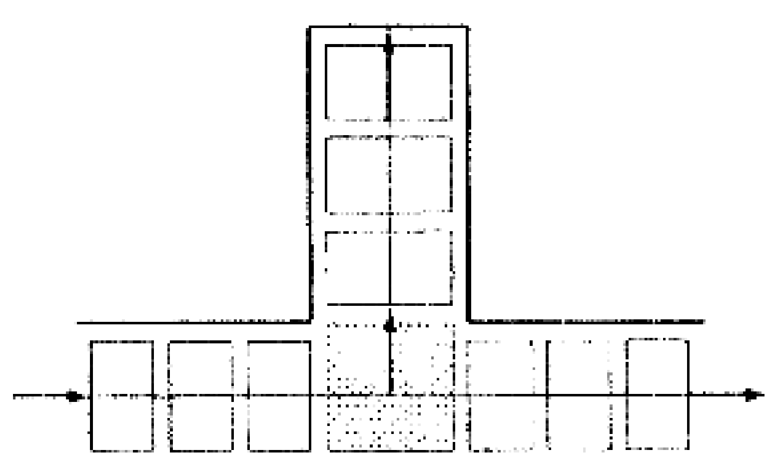

Figure 4

After the silencer there is an exit of finite length, where the waves propagating to the atmospheric interface are reflected back towards the source. These tailpipe reflections have an effect on the transmission loss characteristics of a silencer and may even cause an increase of the original signal.

CALCULATION OF THE ATTENUATION

The attenuation is calculated in terms of the transmission loss, defined as the ratio between the sound power entering and leaving the silencer. If the inlet and outlet ducts have the same cross-sectional area, as it is the case in this study, the ratio of sound powers becomes equal to the ratio of the maximum sound pressure amplitudes. Then, the transmission loss (in dB) is given by

$$TL(\omega) = 10 \log_{10} \left| \frac{C_{+i}(\omega)}{C_{+t}(\omega)} \right|^2$$

where $C_+(\omega)$ is the magnitude of the fluctuating pressure in the positive direction at a frequency ω, and the subscripts i and t refer to incident and transmitted components, respectively. Once the pressure fluctuations before and after the silencer are obtained in the time domain, the positive wave has to be calculated by means of the following expression.

$$C_+(\omega) = p(x,t)\,\frac{tg(\omega t + k x)}{\cos(\omega t - k x)\,tg(\omega t + k x) - sen(\omega t - k x)}$$

which is obtained from the wave equation. The Fast Fourier Transform is applied to these signals in order to obtain the transmission loss at the required frequency.

RESULTS

The described algorithm was applied to several configurations of the three silencers described before. A summary of the results is presented here. Figure 5 shows the time evolution of the pressure at several positions along the duct and the silencer of a Helmholtz resonator for a frequency of 70 Hz. The calculation process requires some time steps until convergence is reached. The lengths before and after the silencer were 0.2 m and 1.5 m respectively, and the volume of the resonator was $9\cdot10^{-3}$ m^3.

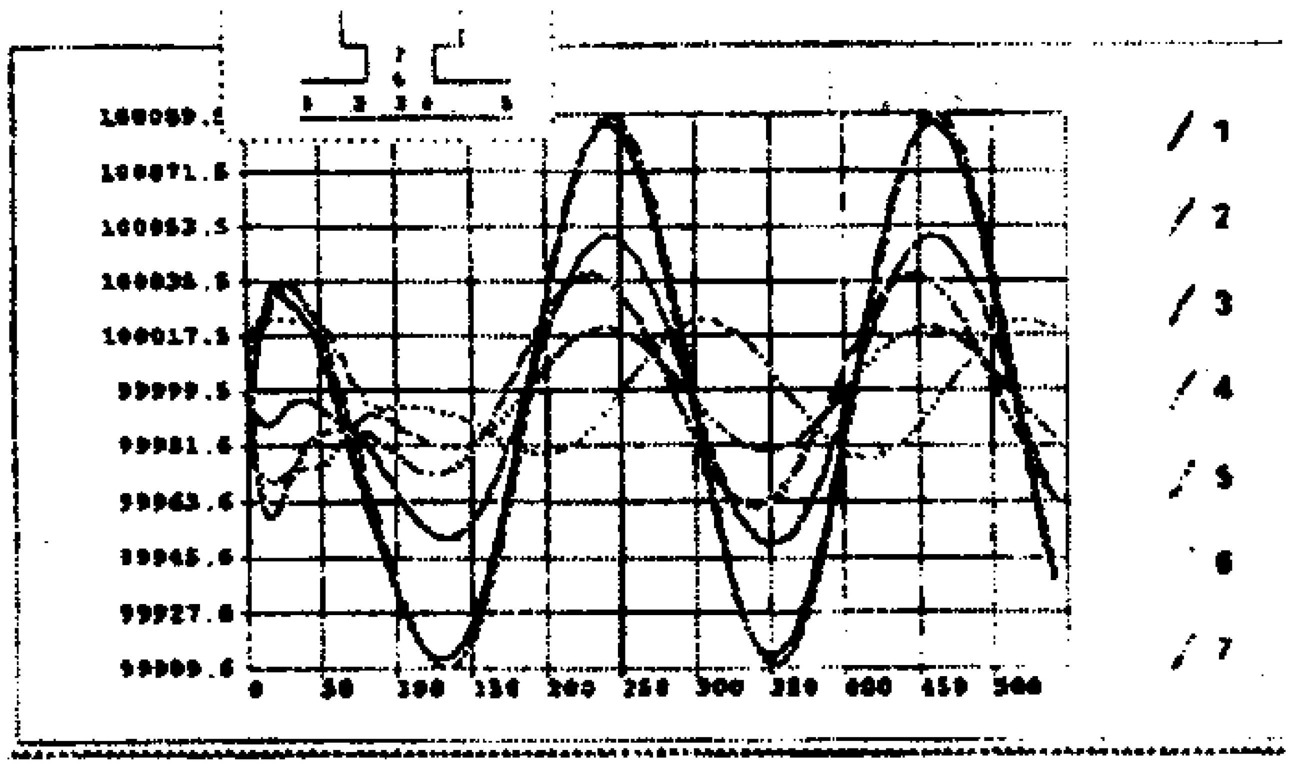

Figure 5

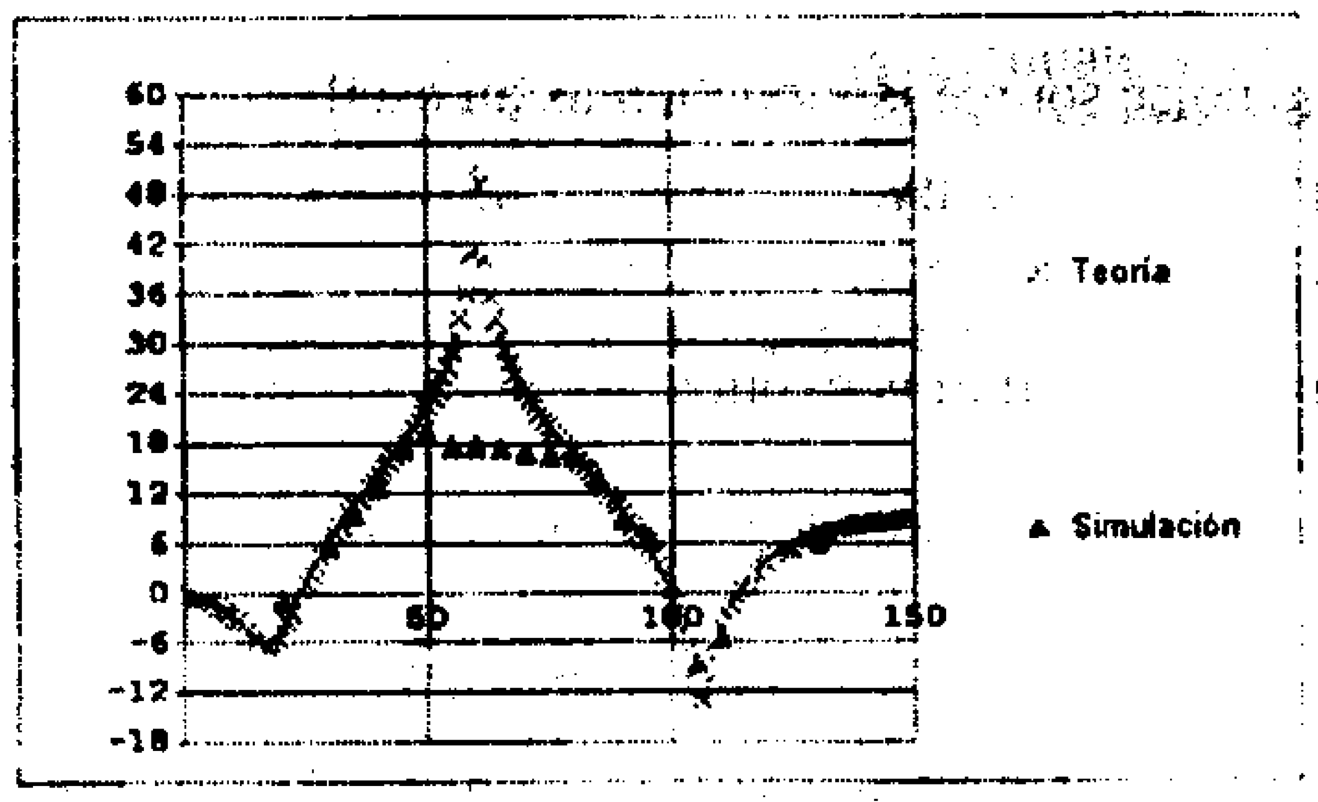

Figure 6

Repeating the process for more frequencies, a relationship is found between the transmission loss and the frequency, which is plotted in figure 6. The calculated data (represented by triangles) are compared with the transmission loss predicted with the acoustic theory applied to this geometry (represented with crosses). Figures 7 and 8 show the same relation for the expansion chamber and the quarter wave resonator. In all cases, good agreement is found between both sets of data throughout the frequency range, except in the resonance region of the Helmholtz resonator, probably due to the three-dimensional effects, not included in the model.

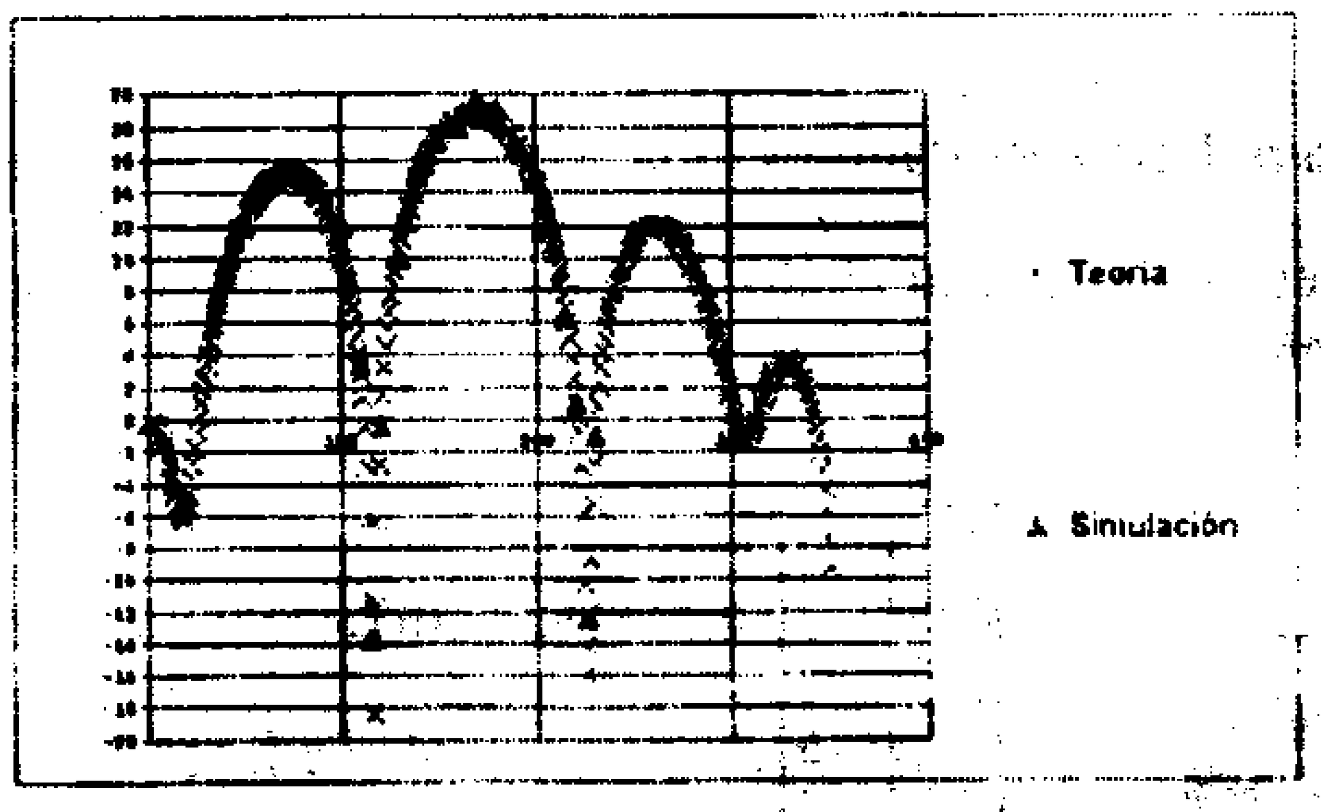

Figure 7

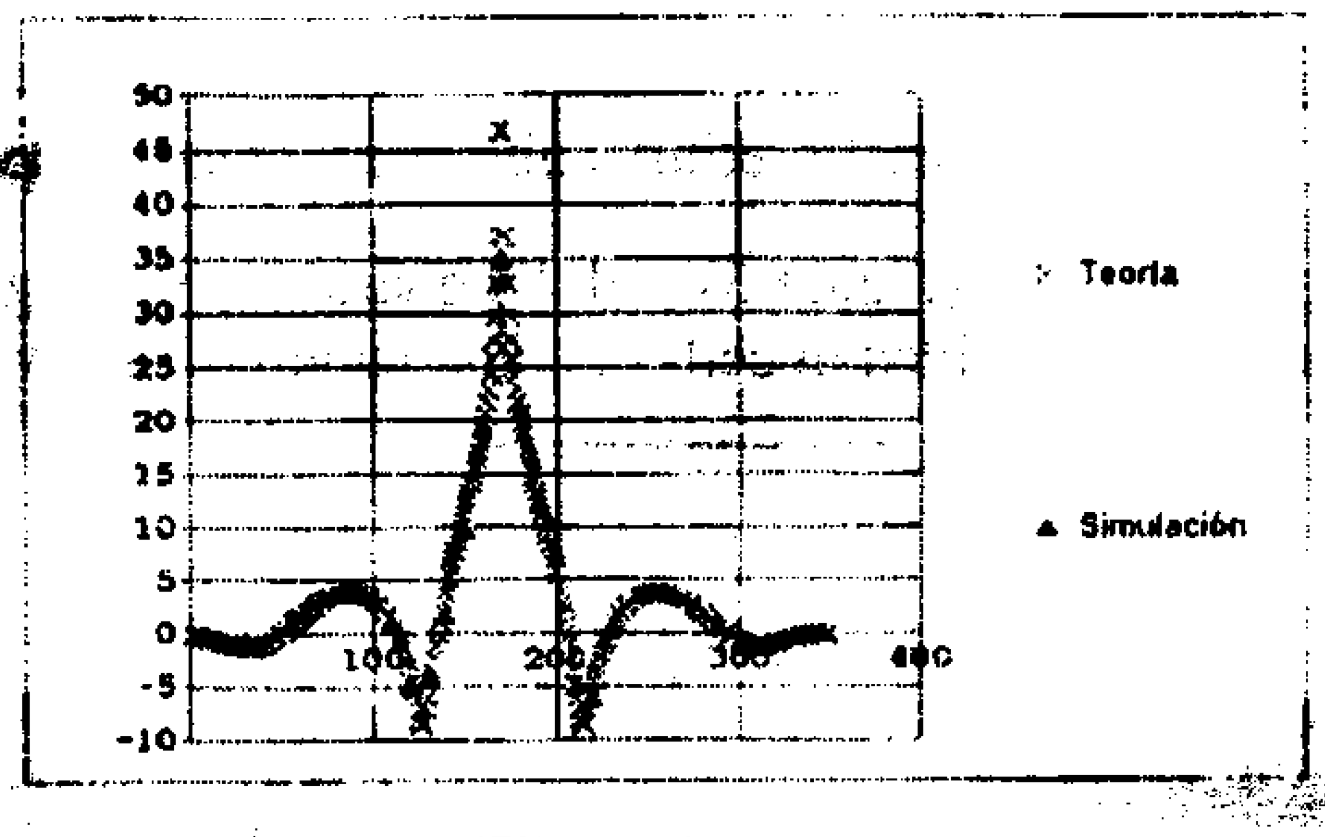

Figure 8

CONCLUSIONS

A one-dimensional finite difference algorithm of the conservation equations for unsteady compressible flow has ebeen developed to study the attenuation of sound in reactive silencers with finite-tailpipe termination. The algorithm was applied to different configurations of reactive silencers and the results showed a remarkable agreement with the theoretical transmission loss predicted with the acoustic theory. This model can also be extended to a wide range of applications, such as complex geometries, non-linear flow phenomena and entire engine intake and exhaust systems.

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