THIRD ORDER ELASTIC MODULI MEASUREMENTS IN SOFT SOLIDS USING TRANSIENT ELASTOGRAPHY

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Abstract

Transient elastography is a powerful tool to measure the velocity of low frequency shear waves in soft tissues and thus to determine the second order elastic moduli µ. In this paper, it is shown how transient elastography can also achieve the third-order elastic moduli of an Agar-gelatin based phantom. This method requires velocity measurements of polarized elastic waves measured in a statically stressed isotropic medium. A static uniaxial-stress induces a transverse isotropy in solids. In this special case, the anisotropy is not caused by linear elastic coefficients but by the third order non linear elastic constants. Consequently, the velocity variations of the low frequency (50 Hz) polarized shear waves as function of the applied stress allows one to measure the third order Landau moduli A, B, C. The several orders of magnitude found between these three constants can be justify from the expression of the internal energy.

Introduction

Acoustoelasticity is a well-established technique [1] to experimentally measure third order elastic constants in solids such as metals [2], crystals [3] or rocks [4]. It consists in measuring the velocity of ultrasonic waves in stressed solids. The third order moduli are deduced from the slope of the velocity as function of static hydrostatic pressure or uniaxial stress. So far no such measurements have been made in soft tissue since it has long been considered as liquid-like medium from an ultrasonic point of view. However, like in all solids, shear waves do propagate in soft tissues at low frequency (50 Hz typically) [5,6]. As it is shown in this paper, the velocity of these shear waves are modified if the medium is submitted to a uniaxial-stress (which is the evidence of a deviation from the Hooke's law). In such a medium, the uniaxial-stress induces transverse isotropy [7]. Thus a quantitative evaluation of this transverse isotropy with the technique of transient elastography leads to the measurement of the nonlinear coefficients.

I-Experiment

The experiments presented in the following section are lead in a model of soft tissues: an Agar-gelatin based phantom. A 5 MHz transducer is mounted in the middle of a rod fixed on a vibrator (Brüel&Kjaer, type 4810). The whole system is applied at the surface of the phantom so that the ultrasonic beam is horizontal (Fig.1). A rigid Plexiglas plate is placed on the top and loads can be added to control the uniaxial-stress in the sample. The transducer works as a pulse-echo system and backscattered signals sampled at 50 MHz are stored in a 2Mo memory with a recurrence frequency of typically 3000 Hz. The low frequency pulse (50 Hz central frequency) propagates in the medium as a shear wave and the longitudinal displacements are measured with a cross-correlation algorithm between successive A-scans [8]. This technique is known as transient elastography [9]. On the seismic-like representation of the displacements (Fig. 2) obtained in an unstressed medium, the maximum amplitude of the 50 Hz pulse is 120 µm. The shear wave longitudinal component appears at each depth with a phase delay inversely proportional to its velocity. Actually the velocity ($2.48 m.s^{-1}$) is extracted with a simple phase analysis at the central frequency. It is shown in [10] that the rod source (80 mm long) allows one to generate shear waves with a polarization perpendicular to the rod in the first centimeters. Since the velocity of shear waves in a uniaxial stressed medium depends on its polarization as regard to the stress, the velocity of the shear wave is measured for each amount of stress with the rod in the horizontal and vertical position.

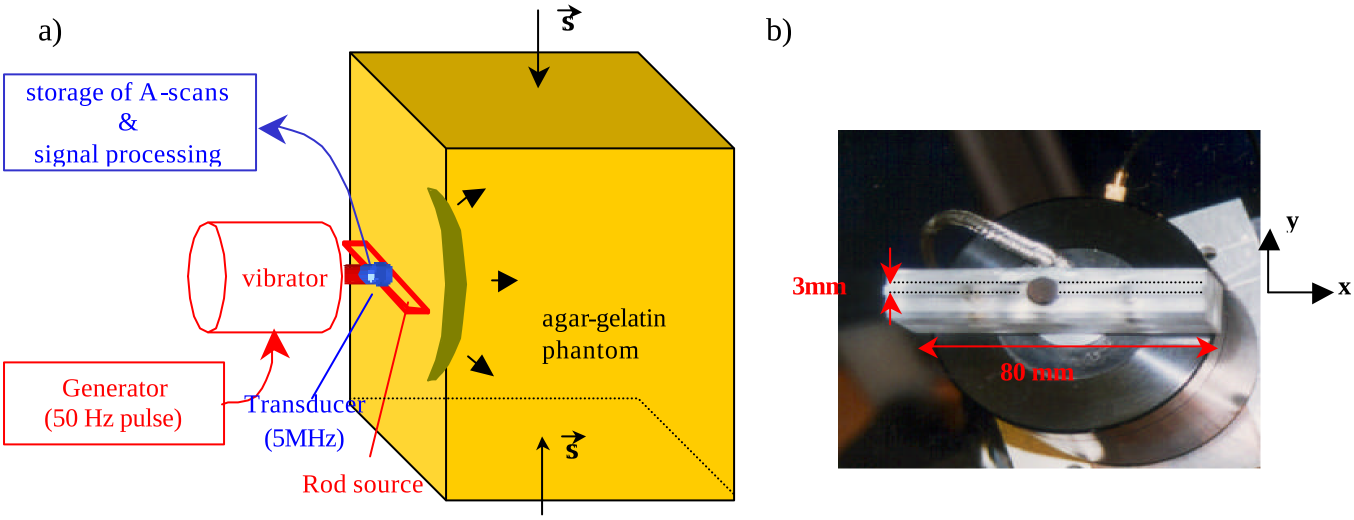

Fig. 1: a) Experimental set-up. A transducer is set in the middle of a rod mounted on a vibrator. A low frequency pulse propagates in the medium and the displacements are computed from the A-scan stored in a memory. b) Picture of the set-up. The transducer (the black disk) is in the middle of a Plexiglas rod and the black vibrator is visible in the back ground.

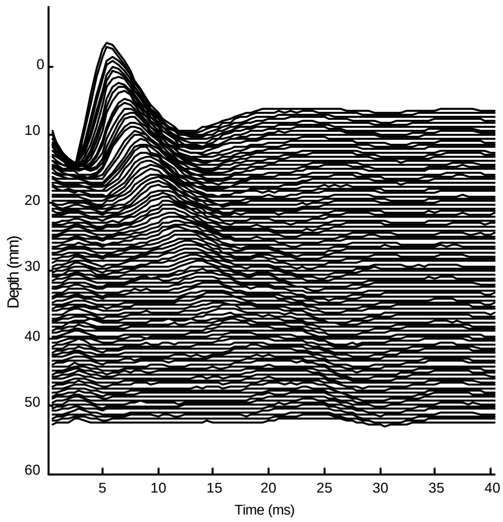

Fig. 2 : Experimental displacement in an Agar-gelatine based phantom. The maximum amplitude of the 50 Hz pulse is 120 µm. A phase analysis gives the velocity of the shear wave: 2.48m.s^{-1}.

II-Theory

Hugues and Kelly [2] have established expressions of the velocity of elastic waves in a uniaxial-stressed solid as function of the second order (Lamé coefficients, l, μ) and the third order moduli (Landau coefficients, *A*, *B*, *C*).

$$r(V_p)^2 = l + 2m - \frac{s}{3l + 2\mu}[-\frac{l}{\mu}A + 2B(1 - \frac{l}{\mu}) + 2C - 4l] \tag{1}$$

$$r(V_s^{//})^2 = \mu - \frac{s}{3l + 2\mu}[\frac{A}{2}(1 + \frac{l}{2\mu}) + B + l + 2\mu] \tag{2}$$

$$r(V_s^{\perp})^2 = \mu - \frac{s}{3l + 2\mu}(\frac{A}{2}(1 - \frac{l + \mu}{2\mu}) + B - 2l) \tag{3}$$

In equations (1, 2, 3), V_P stands for the velocity of the compressional wave, $V_S^{//}$ for the velocity of the shear wave with a polarization parallel to the stress axis and $V_S^{\perp}$ for the velocity of the shear wave with a polarization perpendicular to the stress axis. In an unstressed medium ($\sigma = 0$), one can easily verify that the velocities correspond to an isotropic solid.

III-Results and discussion

On the experimental results of the figure 4, the first observation is that all moduli increases which is a clear evidence of a non linear behavior of the agar-gelatin based phantom. The perpendicular elastic modulus increases by 4% and the parallel modulus by 28%. Now from these slopes and using the set of equation (2, 3), one can deduce the three following values -0,2 MPa, -9.8 GPa for the Landau coefficients A, B respectively. The huge difference between these third order moduli is striking since in more conventional medium such as metal, rocks or crystals they are of the same order of magnitude. Now the last Landau coefficient C can be deduced from results found in the literature. Indeed in [11], with a thermodynamic experimental set-up, Everbach measured the non-linear coefficient $\beta = 3.6$ in gelatin based phantom. Since β is expressed as function of the Landau coefficients as

$$b = -\frac{3}{2} - \frac{A + 3B + C}{r_o c_l^2} = 3.64, \tag{4}$$

we finally obtain C = 18 GPa. The experimental errors on these quantitative evaluation are mainly influenced by diffraction biases cited earlier. Nevertheless, the gap between the first (A) and the two last Landau coefficient (B and C) remains huge. In order to justify this result, we must recall how Landau introduced these coefficients. A third order development of the elastic internal energy in an isotropic solid (equation 5) is expressed as function of the Lamé and the Landau coefficients:

$$e = mu_{ik}^2 + (l + \mu)u_{ll}^2 + \frac{A}{3}u_{ik}u_{il}u_{kl} + Bu_{ik}^2 u_{ll} + \frac{C}{3}u_{ll}^3 \tag{5}$$

In equation 5, the first coefficient μ in front of a shear strain is 10^6 smaller than the coefficient λ in front of the compression strain (one has to keep in mind the order of magnitude of the second order moduli in the Agar-gelatin based phantom, typically $\lambda = 2.2$ GPa, $\mu = 6.5$ KPa). Thus it is not surprising for the third order coefficient A in front of shear strain terms to present a very small value compared to the coefficient B and C in front of terms that contain compression strain. So it appears that soft solids are characterized by a huge difference between both second order and third order moduli.

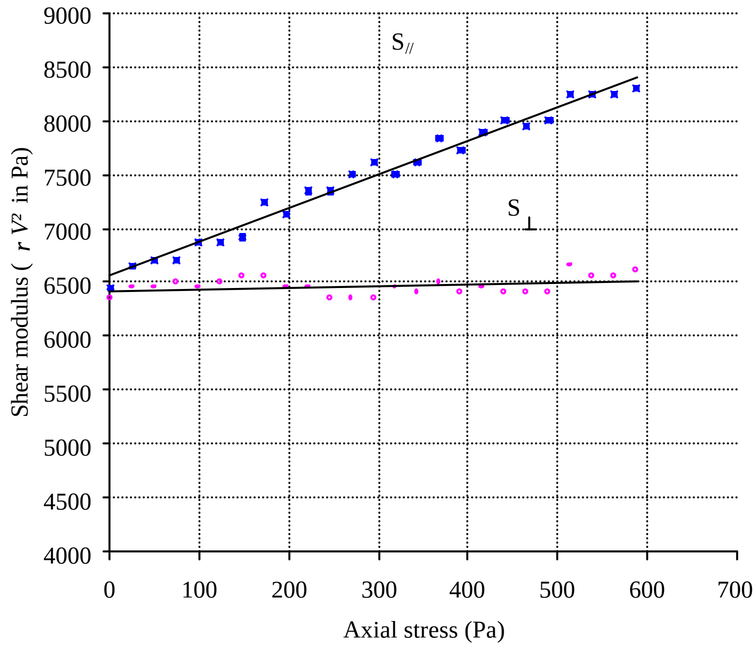

Fig. 3 : Experimental shear and compression moduli as function of the uniaxial stress.

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